THE
ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY
AND ASTRONOMICAL PHYSICS

VOLUME XI JUNE, 1900 NUMBER 5

THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

By James E. Keeler.

The Crossley reflector, at present the largest instrument of its class in America, was made in 1879 by Dr. A. A. Common, of London, in order to carry out, and test by practical observation, certain ideas of his respecting the design of large reflecting telescopes. For the construction of the instrument embodying these ideas, and for some fine astronomical photographs obtained with it, Dr. Common was awarded the gold medal of the Royal Astronomical Society in 1884.

In 1885, Dr. Common, wishing to make a larger telescope on a somewhat similar plan, sold the instrument to Edward Crossley, Esq., F. R. A. S., of Halifax, England. Mr. Crossley provided the telescope with a dome of the usual form, in place of the sliding roof used by its former owner, and made observations with it for some years; but the climate of Halifax not being suitable for the best use of such a telescope, he consented, at the request of Dr. Holden, then Director of the Lick Observatory, to present it to this institution. The funds for transporting the telescope and dome to California, and setting them

up on Mt. Hamilton, were subscribed by friends of the Lick Observatory, for the most part citizens of California. The work was completed, and the telescope housed in a suitable observatory building, in 1895.[1]

On taking charge of the Lick Observatory in 1898, I decided to devote my own observing time to the Crossley reflector, although the whole of my previous experience had been with refracting telescopes. I was more particularly desirous of testing the reflector with my own hands, because such preliminary trials of it as had been made had given rise to somewhat conflicting opinions as to its merits.[2] The result of my experience is given in the following article, which is written chiefly with reference to American readers. If I have taken occasion to point out what I regard as defects in the design or construction of the instrument, I have done so, not from any desire to look a gift horse in the mouth, but in the interest of future improvement, and to make intelligible the circumstances under which the work of the reflector is now being done and will be done hereafter. The most important improvements which have suggested themselves have indeed already been made by Dr. Common himself, in constructing his five-foot telescope. The three-foot reflector is, in spite of numerous idiosyncracies which make its management very different from the comparatively simple manipulation of a refractor, by far the most effective instrument in the Observatory for certain classes of astronomical work. Certainly no one has more reason than I to appreciate the great value of Mr. Crossley's generous gift.

The Crossley dome (Plate VI) is about 350 yards from the main Observatory, at the end of a long rocky spur which extends from the Observatory summit toward the south, and on which are two of the houses occupied by members of the Observatory

[1] For a more complete history of this part of the subject, see Dr. Holden's articles in *Pub. Ast. Soc. Pacific*, 7, 197 *et seq.*, 1895.

[2] The difficulties here referred to, about which a good deal has been written, seem to have had their origin in the fact that it was impossible, at the time of the preliminary trials, to provide the observer with an assistant, while the Crossley reflector is practically unmanageable by a single person.

PLATE VI

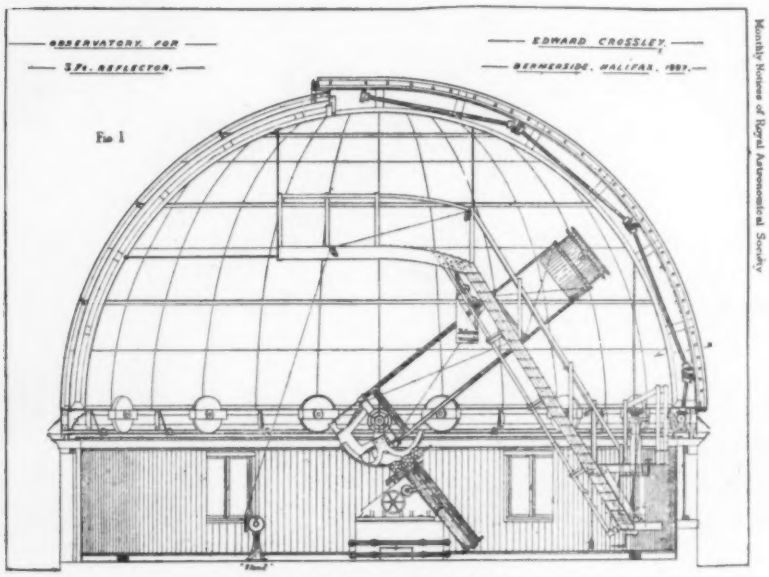
DOME OF THE CROSSLEY REFLECTOR

staff. It is below the level of the lowest reservoir, "Huyghens," which receives the discharge from the hydraulic machinery of the 36-inch refractor, and therefore the water engine furnished by Mr. Crossley for turning the dome cannot be used, unless a new water system — overflow reservoir, pump and windmill — is provided. In this respect a better site would have been a point on the south slope of "Kepler,"—the middle peak of Mt. Hamilton — just above the Huyghens reservoir. No addition to the present water system would then have been needed. The slope of the mountain at this place might cut off the view of the north horizon, but since the telescope cannot be turned below the pole, this would be a matter of no consequence. Water-power for the dome is not, however, really necessary.

The cylindrical walls of the dome, 36¼ feet inside diameter, are double, and provided with ventilators. Opening into the dome, on the left of the entrance, are three small rooms, one of which has been fitted up as a photographic dark room, and another, containing a sidereal clock and a telephone which communicates with the main Observatory, as a study, while the third is used for tools and storage. There is also a small room for the water engine, in case it should be used. The dome is at present supplied with water from only the middle reservoir, Kepler, which is reserved for domestic purposes and is not allowed to pass through the machinery.

The dome itself, 38 feet 9 inches in diameter, is made of sheet-iron plates riveted to iron girders. It also carries the wooden gallery, ladders, and observing platform, which are suspended from it by iron rods. The apparatus for turning the dome consists of a cast-iron circular rack bolted to the lower side of the sole-plate, and a set of gears terminating in a sprocket-wheel, from which hangs an endless rope. As the dome does not turn easily, it has been necessary to multiply the gearing of the mechanism so that one arm's-length pull on the rope moves the dome only about one inch. In some positions of the telescope the dome cannot be moved more than six or eight inches at a time without danger of striking the tube, and

this slowness of motion is then not disadvantageous. It is only when the dome has to be moved through a considerable angle, as in turning to a fresh object, or in photographing some object which passes nearly through the zenith, that the need for a mechanical means of rotation is felt.



THE CROSSLEY REFLECTOR.

The observing slit, 6 feet wide, extends considerably beyond the zenith. It is closed by a double shutter, which is operated by an endless rope. The upper part, within the dome, is also closed by a hood, or shield, which serves to protect the telescope from any water that may find its way through the shutter, and which is rolled back to the north when observations are made near the zenith. I have recently fitted the lower half of the slit with a wind-screen, which has proved to be a most useful addition. It is made of tarpaulin, attached to slats which slide between the two main girders, and is raised or lowered by halliards, which belay to cleats on the north rail of the gallery. A more detailed description of the dome has been given in an

article by Mr. Crossley,[1] from which the reduced figure in Fig. 1[2] has been taken.

The mounting of the three-foot reflector has been very completely described and illustrated by Dr. Common,[3] so that only a very general description need be given here. The most important feature of the mounting is that the telescope tube, instead of being on one side of the polar axis, as in the usual construction, is central, so that the axis of the mirror and the polar axis are in the same line when the telescope is directed to the pole. The declination axis is short, and is supported by a massive goose-neck bolted to the upper end of the polar axis. The mirror is placed just *above* the declination axis. Its weight, and the weight of the whole tube and eye-end, are counterpoised by slabs of lead, placed in two iron boxes between which the goose-neck of the polar axis passes. The great advantage of this arrangement, and the controlling principle of the design, is that the telescope is perfectly free to pass the meridian at all zenith distances. No reversal of the instrument is needed, or is indeed possible.

For long-exposure photography, the advantage above referred to is obvious, but it is attended by certain disadvantages. One of these is that a very much larger dome is required than for the usual form of mounting. Another is the great amount of dead weight which the axes must carry; for the mirror, instead of helping to counterpoise the upper end of the tube, must itself be counterpoised. When anything is attached to the eye-end (and in astrophysical work one is always attaching things to the eye-end of a telescope), from ten to twenty times as much weight must be placed in the counterpoise boxes below the declination axis. Where room is to be found for the weights required to counterpoise the Bruce spectrograph, is a problem which I have not yet succeeded in solving.

In his five-foot reflector, Dr. Common has caused the telescope tube to swing between two large ears, which project

[1] *Mon. Not. R. A. S.*, **48**, 316.

[2] Kindly lent by the Astronomical Society of the Pacific.

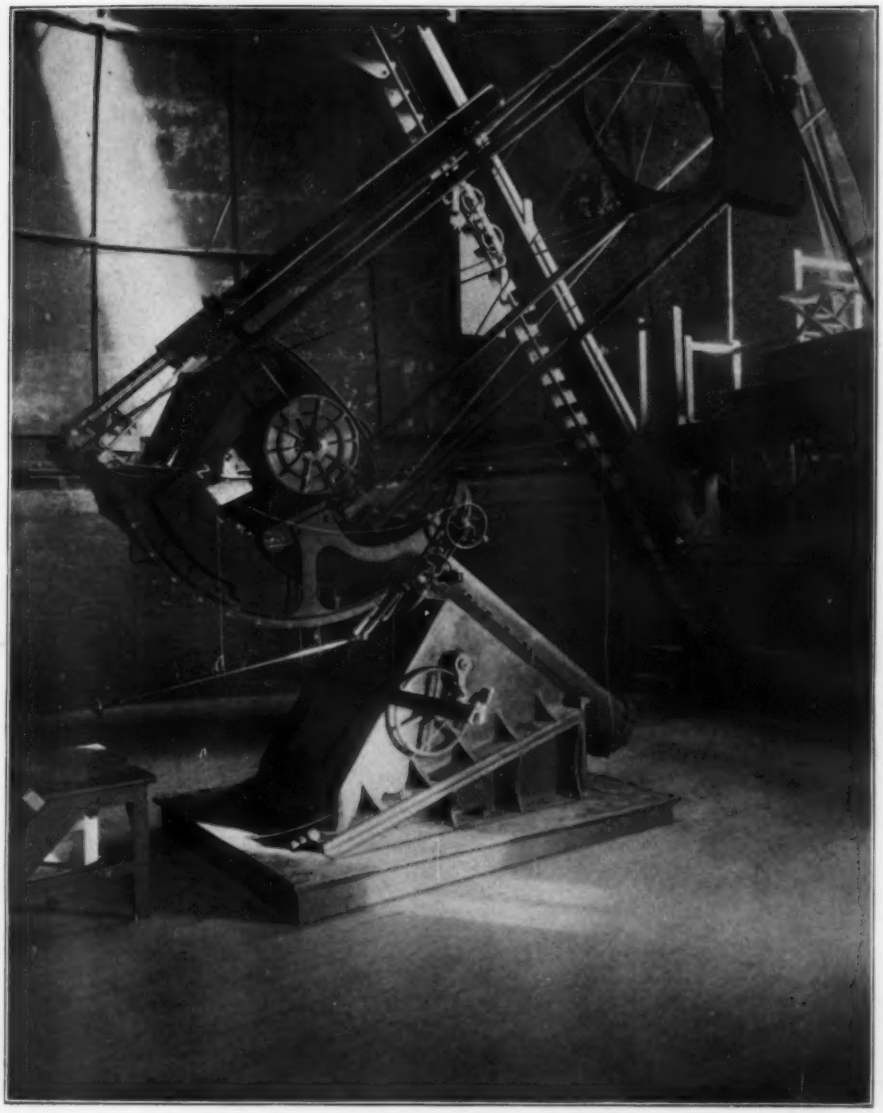
[3] *Mem. R. A. S.*, **46**, 173.

from the upper end of the boiler-like polar axis, the pivots constituting the declination axis being near, but above, the lower end of the tube. The mirror therefore helps to counterpoise the upper end of the tube. This I regard as a distinct improvement. The danger of large masses of metal near the mirror injuring the definition is, in my opinion, imaginary; at least there is no such danger on Mt. Hamilton, where the temperature variations are unusually small. Experience with the Crossley reflector, as well as with the other instruments of the Lick Observatory, shows that the definition depends almost entirely on external conditions.

My first trials of the reflector, as first mounted at the Lick Observatory, showed that the center of motion was inconveniently high. Among other difficulties arising from this circumstance, the spectroscope projected beyond the top of the dome, so that it had to be removed before the shutter could be closed. In July 1898 the pier was therefore cut down two feet. This brought the eye-end down nearly to the level of the gallery rail, where it was at a convenient height for the observer when sitting on a camp-stool, and it made all parts of the mounting more accessible. Toward the north and south, the range of the telescope, being limited in these directions by the construction of the mounting, was not affected by the change, but the telescope cannot now be used at such low altitudes as formerly, near the east and west points of the horizon. The only occasion likely to call for the use of the reflector in these positions is the appearance of a large comet near the Sun, and after some consideration, I decided to sacrifice these chances for the sake of increasing the general usefulness of the instrument. Except in rare cases, all observations are made within three hours of the meridian.

To adapt the mounting to the latitude of Mt. Hamilton, a wedge-shaped casting, shown in Plate VII, had been provided, but through some error, arising probably from the fact that the telescope had been used in two different latitudes in England, the angle of the casting was too great. When the pier was cut down its upper surface was therefore sloped toward the south, in order

PLATE VII



THE CROSSLEY REFLECTOR

to compensate the error in the casting. Plate VII shows the instrument very nearly as it is at the present time.

The polar axis of the Crossley reflector is a long, hollow cylinder, separated by a space of about one eighth of an inch from its concentric casing. The idea was to fill this space with mercury, and float the greater part of the thrust of the axis, the function of a small steel pin at the lower end being merely to steady the axis. But this mercury flotation, as applied to the Crossley telescope, is a delusion, as I think Mr. Crossley had already found. The mercury, it is true, relieves the thrust to some extent, but it greatly increases the already enormous side pressure on the steel pin at the bottom, thus creating a much greater evil than the one it is intended to remedy. The workmen who set up the mounting inform me that the small bearing at the lower end of the polar axis is badly worn, as I should expect it to be. Instead of putting mercury into the space intended for it, I have therefore poured in a pint or so of oil, to keep the lower bearing lubricated. For the reasons indicated above, the force required to move the telescope in right ascension is perhaps five times greater than it should be. The lower end of the polar axis ought to be fitted with ball bearings to take the thrust, and with a pair of friction wheels on top; but it would be difficult to make these changes now. It should be observed that the disadvantages of the mercury flotation are considerably greater at Mt. Hamilton than at the latitude for which the telescope was designed.

As already stated above, the range of the telescope is limited on the south by the construction of the mounting. The greatest southern declination which can be observed is 25°. In England this would doubtless mark the limit set by atmospheric conditions, but at Mt. Hamilton it would be easy to photograph objects 15° farther south, if the telescope could be pointed to them.

The original driving-clock having proved to be inefficient, at least without an electric control, a new and powerful driving-clock was made by the Observatory instrument maker, from

designs by Professor Hussey. In its general plan it is like that of the 36-inch refractor. The winding apparatus, contained in the large casting of the original mounting, has no maintaining power, and cannot easily be fitted with one. The clock could in no case be wound during a photographic exposure, on account of the tremors attending the operation, but it would be somewhat more convenient to have the stars remain on the plate during the winding. With a little practice, however, one can wind the clock without actually stopping it, though the object must afterwards be brought back to its place by means of the slow motion in right ascension.

Two finders have recently been fitted to the Crossley reflector. One has an object-glass of four inches aperture and eight feet six inches focal length, with a field of about $1^\circ\ 2'$, which is very nearly the photographic field of the main telescope. Its standards are bolted to one of the corner tubes of the reflector, as shown in Plate VII. The other finder has a three-inch objective and a large field. It had not been mounted when the photograph for the plate was made.

When a telescope is used for photographing objects near the pole, with long exposures, the polar axis must be quite accurately adjusted, for otherwise the centers of motion of the stars and of the telescope will not agree, and the star images will be distorted. It is true that with a double-slide plate-holder, like the one used with the Crossley reflector, one star — namely the guiding star — is forced to remain in a fixed position with respect to the plate; but the differential motion of the other stars causes them to describe short arcs, or trails, around this star as a center. A considerable part of the spring of 1899 was spent in efforts to perfect the adjustment of the polar axis, an operation which, on account of the peculiar form of the mounting, offers unusual difficulties.

In the first plan which was tried, the reflector was used as a transit instrument. The inclination of the declination axis was determined with a hanging level which had been provided by Mr. Crossley, the hour circle and polar axis being very firmly

clamped. The clock correction being known from the records kept at the Observatory, the collimation and azimuth constants were found by the usual formulæ. This method failed to give satisfactory results, and it was found later that the declination and polar axis were not exactly at right angles.

There is only one part of the sky on which the telescope can be reversed; namely, the pole. A method which promised well, and on which some time was spent, consists in photographing the pole (the declination axis being horizontal) by allowing the stars near it to trail for ten or fifteen minutes, then turning the polar axis 180° and photographing the pole again on the same plate. Half the distance between the images gives the error of the polar axis, which if the plate is properly oriented, is easily resolved into horizontal and vertical components; while the distance of each image from the center of the plate is this error increased or diminished by twice the deviation of the telescope axis. In this case the vertical component depends upon the reading of the declination circle, and the horizontal component gives the error of collimation. This method failed, however, to give consistent results, mainly on account of instability of the mirror, and was abandoned.

The use of the large mirror for purposes of adjustment was finally given up, and the axis was adjusted by observations of *Polaris* with the long finder, in the usual manner. In order to reach the star at lower culmination the finder tube had to be thrown out of parallelism with the main telescope.

The base-plate having no definite center of rotation in azimuth, and the wedges and crowbars used for moving it being uncertain in their action, a watch telescope, provided with a micrometer eyepiece, was firmly secured to the mounting throughout these operations, in such manner that a mark on the southern horizon could be observed through one of the windows of the dome. The errors of the polar axis were finally reduced to within the limits of error of observation.

The movable hour circle and driving wheel of the Crossley reflector has two sets of graduations. The driving screw having

been thrown out of gear, the circle is turned until the outer vernier indicates the sidereal time, whereupon the driving screw is thrown into gear again. The inner vernier is then set to the right ascension of the object which it is desired to observe. As an inconsistency, of minor importance, in the design of the mounting, I may note that the slow motion in right ascension changes the reading of the outer vernier instead of that of the inner one. In practice, however, no inconvenience is caused by this construction.

In the early experiments and photographic work with the Crossley telescope, irregularities in driving were a source of great annoyance. Dr. Roberts, in laying down the conditions which should be fulfilled by a good photographic telescope, says that a star should remain bisected by a thread in the eyepiece for two minutes at a time. The Crossley telescope was so far from fulfilling this condition that a star would not keep its place for two consecutive seconds; and the greatest alertness on the part of the observer did not suffice to ensure round star images on a photographic plate. It was obvious that the fault did not lie with the driving clock; in fact, many of the sudden jumps in right ascension, if explained in this way, would have required the clock to run backward; nevertheless the clock was tested by causing its revolutions to be recorded on a chronograph at the main Observatory, together with the beats of one of the standard clocks. For this purpose a break-circuit attachment was made by Mr. Palmer. The errors of the clock were in this way found to be quite small.

The principal source of the irregularities was found in the concealed upper differential wheel of the Grubb slow motion. This wheel turned with uncertain friction, sometimes rotating on its axis, and sometimes remaining at rest. After it was checked the driving was much better, and was still farther improved by repairing some defective parts of the train. Small irregularities still remain. They seem to be partly due to inaccuracies in the cutting of the gears, or of the teeth of the large driving wheel, and partly to the springing of the various parts, due to the very

considerable friction of the polar axis in its bearings. The remaining irregularities are so small, however, that they are easily corrected by the screws of the sliding plate-holder, and with reasonable attention on the part of the observer, round star images are obtained with exposures of four hours' duration.

The large mirror, the most important part of the telescope, has an aperture of three feet, and a focal length of 17 feet 6.1 inches. It was made by Mr. Calver.* Its figure is excellent. On cutting off the cone of rays from a star, by a knife-edge at the focus, according to the method of Foucault, the illumination of the mirror is very uniform, while the star disks as seen in an ordinary eyepiece are small and almost perfectly round. They are not, I think, quite so good as the images seen with a large refractor; still, they are very good indeed, as the following observations of double stars, made recently for this purpose, will show.

Several close double stars were examined on the night of April 17, 1900, with a power of 620. The seeing was four on a scale of five. The magnitudes and distances of the components, as given in the table, are from recent observations by Professor Hussey with the 36-inch refractor.

Star	Mag.	*d.*	Result of Obs.
0Σ 208 (*φ Urs. Maj.*)	5.0, 5.5	0".35	Not resolved; too bright.
0Σ 249, AB	7.2, 8.0	0.54	Easily resolved.
0Σ 250	7.7, 8.0	0.44	Resolved.
0Σ 267	8.0, 8.2	0.30	Just resolved at best moments.

Although the theoretical limit of resolution for a three-foot aperture is not reached in these observations, I do not think the mirror can do any better.

The small mirror, or flat, at the upper end of the tube, is circular, the diameter being nine inches. Its projection on the plane of the photographic plate is therefore elliptical; but the projection of the mirror and its cell on the plane of the great mirror is very nearly circular.

The small mirror, acting as a central stop, has the effect of diminishing the size of the central disk of the diffraction pattern,

*and re-figured by Ritchey.

at the expense of an increase in the brightness of the system of rings. To this effect may be due, in part, the inferiority of the reflector for resolving bright doubles, as compared with a refractor of the same aperture. For photographic purposes, it is evident that the mirror is practically perfect.

The upper end of the tube can be rotated, carrying with it the flat and the eye-end. Whenever the position is changed, the mirrors have to be re-collimated. In practice it is seldom necessary to touch the adjusting screws of the mirrors themselves. The adjustment is effected by means of clamping and butting screws on the eye-end, and a change of the line of collimation, with respect to the finders and the circles, is avoided. The operation is generally referred to, however, as an adjustment of the mirrors.

For adjusting the mirrors there are two collimators. One of these is of the form devised by Mr. Crossley.[1] It is very convenient in use, and is sufficiently accurate for the adjustment of the eye-end when the telescope is used for photographic purposes, inasmuch as the exact place where the axis of the large mirror cuts the photographic plate is not then a matter of great importance, so long as it is near the center. Moreover, as stated farther below, the direction of the axis changes during a long exposure. The other collimator is of a form originally due, I think, to Dr. Johnstone Stoney. It consists of a small telescope, which fits the draw-tube at the eye-end. In the focus of the eyepiece are, instead of cross-wires, two adjustable terminals, between which an electric spark can be passed, generated by a small induction machine, like a replenisher, held in the observer's hand. The terminals are at such a distance inside the principal focus of the objective, that the light from the spark, after reflection from the flat, appears to proceed from the center of curvature of the large mirror. The rays are therefore reflected back normally, and form an image of the spark which, when the mirrors are in perfect adjustment, coincides with the spark itself. The precision of this method is very great. It is

[1] *Mon. Not. R. A. S.*, 48, 280, 1888.

in fact out of proportion to the degree of refinement attained in other adjustments of the reflector, for a slight pressure of the hand on the draw-tube, or movement of the telescope to a different altitude, instantly destroys the perfection of the adjustment. I have provided these collimators with an adapter which fits the photographic apparatus, so that one can adjust the mirrors without having to remove this apparatus and substitute for it the ordinary eye-end carrying the eyepieces.

For visual observation the Crossley telescope is provided with seven eyepieces, with powers ranging from 620 downward. The lowest power is only 60, and consequently utilizes only 12 inches of the mirror, 9 of which are covered by the central flat. It is therefore of little value, except for finding purposes. The next lowest power utilizes 28 inches of the mirror. The other eyepieces call for no remark.

But, while the Crossley reflector would doubtless be serviceable for various kinds of visual observations, its photographic applications are regarded as having the most importance, and have been chiefly considered in deciding upon the different changes and improvements which have been made.

The interior of the dome is lighted at night by a large lamp, which is enclosed in a suitable box or lantern, fitted with panes of red glass, and mounted on a portable stand. In order to diffuse the light in the lower part of the dome, where most of the assistant's work is done, the walls are painted bright red; while to prevent reflected light from reaching the photographic plate, the inner surface of the dome itself, the mounting, and the ladders and gallery are painted dead black. The observer is therefore in comparative darkness, and not the slightest fogging of the plate, from the red light below, is produced during a four-hours' exposure. On the few occasions when orthochromatic plates are used, the lamp need not be lighted.

Experiments have shown that the fogging of the photographic plate, during a long exposure, is entirely due to diffuse light from the sky, and is therefore unavoidable. For this reason the cloth curtains which lace to the corners of the telescope

tube, enclosing it and shutting out light from the lower part of the dome, have not been used, since their only effect would be to catch the wind and cause vibrations of the telescope. They would probably have little effect on the definition, and at any rate could not be expected to improve it.

For photographing stars and nebulæ the Crossley reflector is provided with a double-slide plate-holder, of the form invented by Dr. Common.[1] This apparatus, which had suffered considerably in transportation, and from general wear and tear, was thoroughly overhauled by the Observatory instrument-maker. The plates were straightened and the slides refitted. A spring was introduced to oppose the right ascension screw and take up the lost motion — the most annoying defect that such a piece of apparatus can have — and various other improvements were made, as the necessity for them became apparent. They are described in detail farther below.

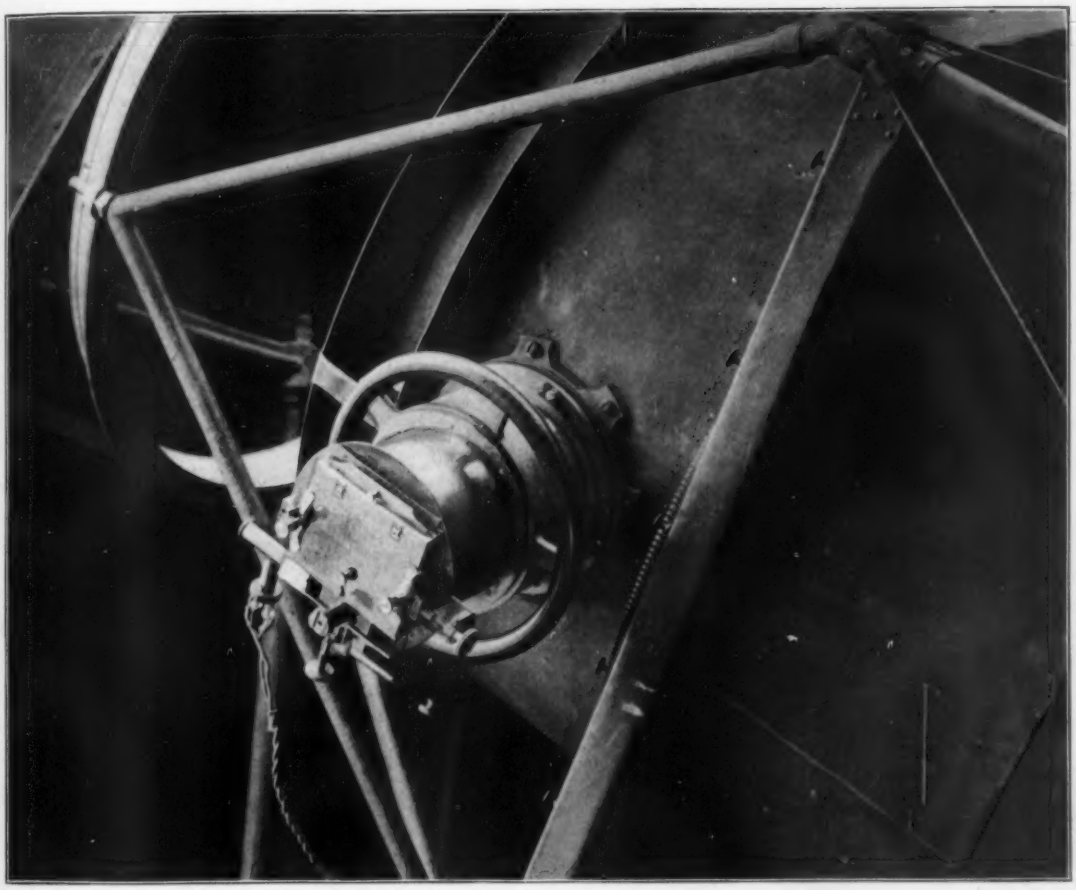
The present appearance of the eye-end is shown in Plate VIII. The plate-holder is there shown, however, on one side of the tube, and its longer side is parallel to the axis of the telescope. This is not a good position for the eye-end, except for short exposures. In practice, the eye-end is always placed on the north or south side of the tube, according as the object photographed is north or south of the zenith. The right ascension slide is then always at right angles to the telescope axis, and the eye-end cannot get into an inaccessible position during a long exposure.

As the original wooden plate-holders were warped, and could not be depended upon to remain in the same position for several hours at a time, they were replaced by new ones of metal, and clamping screws were added, to hold them firmly in place. The heads of these screws are shown in the plate, between the springs which press the plate-holder against its bed.

To illuminate the cross-wires of the guiding eyepiece, a small electric lamp is used, the current for which is brought

[1] *Mon. Not. R. A. S.*, 49, 297. The construction here described is not followed exactly in the Crossley apparatus. The guiding eyepiece slides freely when not held by a clamp. Pin-holes for preventing fogging are unnecessary when red light is used.

PLATE VIII



DOUBLE-SLIDE PLATE HOLDER OF THE CROSSLEY REFLECTOR

down from the storage battery at the main Observatory. The coarse wires have been replaced by spider's webs,[1] and reflectors have been introduced, to illuminate the declination thread. A collimating lens, placed at its principal focal distance from the incandescent filament of the lamp, makes the illumination of the wires nearly independent of their position on the slide, and a piece of red glass, close to the lens, effectually removes all danger of fogging the plate. The light is varied to suit the requirements of observation by rotating the reflector which throws the light in the direction of the eyepiece.

In long exposures it is important for the observer to know at any moment the position of the plate with reference to its central or zero position. For this purpose scales with indexes are attached to both slides; but as they cannot be seen in the dark, and, even if illuminated with red light, could not be read without removing the eye from the guiding eyepiece, I have added two short pins, one of which is attached to the lower side of the right ascension slide, and the other to its guide, so that the points coincide when the scale reads zero. These pins can be felt by the fingers, and with a little practice the observer can tell very closely how far the plate is from its central position. It would not be a very difficult matter to improve on this contrivance, say by placing an illuminated scale, capable of independent adjustment, in the field of the eyepiece, but the pins answer every purpose. The declination slide is changed so little that no means for indicating its position are necessary.

In this apparatus, as originally constructed, the cross-wires of the guiding eyepiece were exactly in the plane of the photographic plate. The earlier observations made with the Crossley reflector on Mt. Hamilton showed that this is not the best position of the cross-wires. The image of a star in the guiding eyepiece, which, when in the middle of its slide, is nearly three inches from the axis of the mirror, is not round, and its shape

[1] It so happens that the tension of the vertical thread is such that it begins to slacken when the temperature falls to within about 2° of the dew point. The thread thus forms an excellent hygrometer, which is constantly under the eye of the observer. When the thread becomes slack, it is time to cover the mirrors.

varies as the eyepiece is pushed in or drawn out. In the plane of the photographic plate (assumed to be accurately in focus), it is a crescent, with the convex side directed toward the center of the plate. This form of image is not suitable for accurate guiding. Outside this position the image changes to an arrow-head, the point of which is directed toward the axis, and this image can be very accurately bisected by the right ascension thread. As the construction of the apparatus did not allow the plane of the cross-wires to be changed, the wooden bed of the plate-holder was cut down, so as to bring the wires and the plate into the proper relative positions.

After some further experience with the instrument, still another change was made in this adjustment. It was found that the focus often changed very perceptibly during a long exposure, and while the arrow-head image above described was suitable for guiding purposes, its form was not greatly affected by changes of focus. Between the crescent and the arrow-head images there is a transition form, in which two well-defined caustic curves in the aberration pattern intersect at an acute angle. The intersection of these caustics offers an excellent mark for the cross-wires, and is at the same time very sensitive to changes of focus, which cause it to travel up or down in the general pattern. The bed of the plate-holder was therefore raised, by facing it with a brass plate of the proper thickness.

Why the focus of the telescope should change during a long exposure is not quite clear. The change is much too great to be accounted for by expansion and contraction of the rods forming the tube, following changes of temperature, while a simple geometrical construction shows that a drooping of the upper end of the tube, increasing the distance of the plate from the (unreflected) axis of the mirror, cannot displace the focus in a direction normal to the plate, if it is assumed that the field is flat. The observed effect is probably due to the fact that the focal surface is not flat, but curved. During a long exposure, the observer keeps the guiding star, and therefore, very approximately, all other stars, in the same positions relatively to the

plate; but he has no control over the position of the axis of the mirror, which, by changes of flexure, wanders irregularly over the field. The position of maximum curvature, therefore, also varies, and with it the focus of the guiding star relatively to the cross-wires, where the focal surface is considerably inclined to the field of view. It is certain that the focus does change considerably, whatever the cause may be, and that the best photographic star images are obtained by keeping the focus of the guiding star unchanged during the exposures. This is done by turning the focusing screw of the eye-end.

In making the photographs of nebulæ for which the Crossley telescope is at present regularly employed, it was at first our practice to adjust the driving-clock as accurately as possible to a sidereal rate, and then, when the star had drifted too far from its original position, on account of changes of rate or of flexure, to bring it back by the right ascension slow motion, the observer either closing the slide of the plate-holder or following the motion of the star as best he could with the right-ascension screw. Lately a more satisfactory method, suggested by Mr. Palmer, has been employed. The slow motion in right ascension is of Grubb's form,[1] and the telescope has two slightly different rates, according to whether the loose wheel is stopped or allowed to turn freely. The driving-clock is adjusted so that one of these rates is too fast, the other too slow. At the beginning of an exposure the wheel is, say, unclamped, and the guiding star begins to drift very slowly toward the left, the observer following it with the screw of the plate-holder. When it has drifted far enough, as indicated by the pins mentioned further above, the wheel is clamped. The star then reverses its motion and begins to drift toward the right; and so on throughout the exposure. The advantages of this method over the one previously employed are, that the star never has to be moved by the slow motion of the telescope, and that its general drift is in a known direction, so that its movements can be anticipated by the observer. In this way photographs are obtained, with four

[1] *Mon. Not. R. A. S.*, **48**, 352.

hours' exposure, on which the smallest star disks are almost perfectly round near the center of the plate, and from 2″ to 3″ in diameter.

The star images are practically round over a field at least 1 inch or 16′ in diameter. Farther from the center they become parabolic, but they are quite good over the entire plate, 3¼ by 4¼ inches.

From these statements it will be seen that small irregularities in driving no longer present any difficulties. But certain irregular motions of the image still take place occasionally, and so far it has not been possible entirely to prevent their occurrence.

It was found that the declination clamp (the long slow-motion handle attached to which is shown in Plate VII) was not sufficiently powerful to hold the telescope firmly during a long exposure. A screw clamp was therefore added, which forces the toothed-declination sector strongly against an iron block just behind it, thus restoring, I think, the original arrangement of the declination clamp as designed by Dr. Common. This clamp holds the tube very firmly.

The irregularities to which I have referred consist in sudden and unexpected jumps of the image, which always occur some time after the telescope has passed the meridian. These jumps are sometimes quite large—as much as one sixteenth of an inch or 1′. They are due to two causes; flexure of the tube, and sliding of the mirror on its bed. When the jump is due to sudden changes of flexure, the image moves very quickly, and vibrates before it comes to rest in its new position, and at the same time there is often heard a slight ringing sound from the tension rods of the tube. There seems to be no remedy for the sudden motions of this class. The tension rods are set up as tightly as possible without endangering the threads at their ends or buckling the large corner tubes. A round telescope tube, made of spirally-wound steel ribbon riveted at the crossings, would probably be better than the square tube now in use.

Jumps due to shifting of the mirror are characterized by a gentle, gliding motion. They can be remedied, in part, at least,

by tightening the copper bands which pass around the circumference of the mirror within its cell. This will be done the next time the mirror is resilvered.

All that the observer can do when a jump occurs is to bring back the image as quickly as possible to the intersection of the cross-wires. If all the stars on the plate are faint, no effect will be produced on the photograph; but stars of the eighth magnitude or brighter will leave short trails. The nebula, if there is one on the plate, will, of course, be unaffected.

Before beginning an exposure the focus is adjusted by means of a high-power positive eyepiece. An old negative, from which the film has been partially scraped, is placed in one of the plate-holders, and the film is brought into the common focus of the eyepiece and the great mirror. The appearance of the guiding star, which varies somewhat with the position of the guiding eyepiece on its slide, is then carefully noted, and is kept constant during the exposure by turning, when necessary, the focusing screw of the eye-end. For preliminary adjustments a ground-glass screen is often convenient. On it all the *DM.* stars, and even considerably fainter ones, as well as the nebulæ of Herschel's Class I, are easily visible without a lens.

Plates are backed, not more than a day or two before use, with Carbutt's "Columbian backing," which is an excellent preparation for this purpose. During the exposure the observer and assistant exchange places every half hour, thereby greatly relieving the tediousness of the work, though two exposures of four hours each, in one night, have proved to be too fatiguing for general practice. At the end of the first two hours it is necessary to close the slide and wind the clock.

The brightness of the guiding star is a matter of some importance. If the star is too bright, its glare is annoying; if it is too faint, the effort to see it strains the eye, and changes of focus are not easily recognized. A star of the ninth magnitude is about right. In most cases a suitable star can be found without difficulty.

In such an apparatus as that described above, the amount by which the plate may be allowed to depart from its zero position is subject to a limitation which has not, I think, been pointed out, although it is sufficiently obvious when one's attention has been called to it. It depends upon the fact that the plate necessarily moves as a whole, in a straight line which is tangent to a great circle of the sphere, while the stars move on small circles around the pole. The compensation for drift, when the plate is moved, is therefore exact at the equator only.

Let the guiding star have the declination δ_1, and let a star on the upper edge of the plate (which, when the telescope is north of the zenith, and the eye-end is on the north side of the telescope, will be the southern edge), have the declination δ_2. Then if the guiding star is allowed to drift from its zero position through the distance d, the other star will drift through the distance $d\,\frac{\cos\,\delta_2}{\cos\,\delta_1}$. If the guiding star is followed by turning the right ascension screw, the upper edge of the plate, as well as the guiding eyepiece, will be moved through the distance d. Hence there will be produced an elongation of the upper star, represented by

$$e = d\left(\frac{\cos\,\delta_2}{\cos\,\delta_1} - 1\right)$$

$$\text{from which} \quad d = \frac{e\,\cos\,\delta_1}{\cos\,\delta_2 - \cos\,\delta_1}\,.$$

Now, in the Crossley reflector, the upper edge of the plate and the guiding eyepiece are just about $3\frac{2}{3}$ inches, or 1°, apart. If e is given, the above formula serves to determine the maximum range of the slide for different positions of the telescope.

It has been stated farther above that the smallest star disks, on a good photograph, are sometimes not more than 2″ in diameter, or in a linear measure, about $\frac{1}{20}$ mm. An elongation of this amount is therefore perceptible. There are many nebulæ in high northern declinations, and there are several particularly fine ones in about + 70°. If, therefore, we take $\delta_2 = 70°$, $\delta_1 = 71°$, $e = 0.05$, and substitute these values, we find $d = 1.0$ mm, which

is the greatest permissible range of the plate in photographing these nebulæ. Before I realized the stringency of this requirement, by making the above simple computation, I spoiled several otherwise fine negatives by allowing the plate to get too far from the center, thus producing elongated star images.

There is a corresponding elongation in declination, the amount of which can be determined by an adaptation of the formula for reduction to the meridian, but it is practically insensible.

On account of the short focal length of the three-foot mirror, the photographic resolving power of the telescope is much below its optical resolving power. For this reason the photographic images are less sensitive to conditions affecting the seeing than the visual images. On the finest nights the delicate tracery of bright lines or caustic curves in the guiding star is as clear and distinct as in a printed pattern. When the seeing is only fair these delicate details are lost, and only the general form of the image, with its two principal caustics, is seen. A photograph taken on such a night is not, however, perceptibly inferior to one taken when the seeing is perfect. When, however, the image is so blurred that its general form is barely distinguishable, the photographic star disks are likewise blurred and enlarged, and on such nights photographic work is not attempted.

The foregoing account of the small changes which have been made in the Crossley telescope and its accessories may appear to be unnecessarily detailed, yet these small changes have greatly increased the practical efficiency of the instrument, and therefore, small as they are, they are important. Particularly with an instrument of this character, the difference between poor and good results lies in the observance of just such small details as I have described.

At present the Crossley reflector is being used for photographing nebulæ, for which purpose it is very effective. Some nebulæ and clusters, like the great nebula in *Andromeda* and the *Pleiades*, are too large for its plate ($3\frac{1}{4} \times 4\frac{1}{4}$ in.), but the great majority of nebulæ are very much smaller, having a length

of only a few minutes of arc, and a large-scale photograph is required to show them satisfactorily. It is particularly important to have the images of the involved stars as small as they can be made.

Many nebulæ of Herschel's I and II classes are so bright that fairly good photographs can be obtained with exposures of from one to two hours; but the results obtained with full light-action are so superior to these, that longer exposures of $3\frac{1}{2}$ or 4 hours are always preferred. In some exceptional cases, exposures of only a few minutes are sufficient. The amount of detail shown, even in the case of very small nebulæ, is surprising. It is an interesting fact that these photographs confirm (in some cases for the first time) many of the visual observations made with the six-foot reflector of the Earl of Rosse.

Incidentally, in making these photographs, great numbers of new nebulæ have been discovered. The largest number that I have found on any one plate is thirty-one. Eight or ten is not an uncommon number, and few photographs have been obtained which do not reveal the existence of three or four A catalogue of these new objects will be published in due time.

Some of the results obtained with the Crossley reflector, relating chiefly to particular objects of some special interest, have already been published.[1] The photographs have also per-

[1] The following list includes all papers of interest.

"Photographic Observations of Comet I, 1898 (Brooks), made with the Crossley Reflector of the Lick Observatory," *A. J.* No. 451, **19**, 151; see also *Ap. J.*, **8**, 287.

"The Small Bright Nebula near *Merope*," *Pub. A. S. P.*, **10**, 245.

"On Some Photographs of the Great Nebula in *Orion*, taken by means of the Less Refrangible Rays in its Spectrum," *Ap. J.*, **9**, 133. See also *Pub. A. S. P.*, **11**, 70; *Ap. J.*, **10**, 167; *A. N.*, 3601.

"Small Nebulæ discovered with the Crossley Reflector of the Lick Observatory, *Mon. Not. R. A. S.*, **59**, 537.

"The Ring Nebula in *Lyra*," *Ap. J.*, **10**, 193.

"The Annular Nebula H. IV. 13 in *Cygnus*," *Ap. J.*, **10**, 266; see also *Pub. A. S. P.*, **11**, 177.

"On the Predominance of Spiral Forms among the Nebulæ," *A. N.*, 3601.

"The Distribution of Stars in the Cluster *Messier 13* in *Hercules*," (by H. K. Palmer), *Ap. J.*, **10**, 246.

"The Photographic Efficiency of the Crossley Reflector," *Pub. A. S. P.*, **11**, 199; *Observatory*, **22**, 437.

mitted some wider conclusions to be drawn, which are constantly receiving further confirmation as the work progresses. They may be briefly summarized as follows:

1. Many thousands of unrecorded nebulæ exist in the sky. A conservative estimate places the number within reach of the Crossley reflector at about 120,000. The number of nebulæ in our catalogues is but a small fraction of this.

2. These nebulæ exhibit all gradations of apparent size, from the great nebula in *Andromeda* down to an object which is hardly distinguishable from a faint star disk.

3. Most of these nebulæ have a spiral structure.

To these conclusions I may add another, of more restricted significance, though the evidence in favor of it is not yet complete. Among the objects which have been photographed with the Crossley telescope are most of the "double" nebulæ figured in Sir John Herschel's catalogue (*Phil. Trans.*, 1833, Plate XV). The actual nebulæ, as photographed, have almost no resemblance to the figures. They are, in fact, spirals, sometimes of very beautiful and complex structure; and, in any one of the nebulæ, the secondary nucleus of Herschel's figure is either a part of the spiral approaching the main nucleus in brightness, or it cannot be identified with any real part of the object. The significance of this somewhat destructive conclusion lies in the fact that these figures of Herschel have sometimes been regarded as furnishing analogies for the figures which Poincaré has deduced, from theoretical considerations, as being among the possible forms assumed by a rotating fluid mass; in other words, they have been regarded as illustrating an early stage in the development

"New Nebulæ discovered photographically with the Crossley Reflector of the Lick Observatory," *Mon. Not. R. A. S.*, **60**, 128.

"The Spiral Nebula *H. I. 55 Pegasi*," *Ap. J.*, **11**, 1.

"Photographic Observations of Hind's Variable Nebula in *Taurus*, made with the Crossley Reflector of the Lick Observatory," *Mon. Not. R. A. S.*, **60**, 424.

"Use of the Crossley Reflector for Photographic Measurements of Position," *Pub. A. S. P.*, **12**, 73.

"Discovery and Photographic Observations of a New Asteroid 1899 FD.," *A. N.* 3635.

"Elements of Asteroid 1889 FD.," (by H. K. Palmer), *A. N.*, 3635.

of double star systems. The actual conditions of motion in these particular nebulæ, as indicated by the photographs, are obviously very much more complicated than those considered in the theoretical discussion.

While I must leave to others an estimate of the importance of these conclusions, it seems to me that they have a very direct bearing on many, if not all, questions concerning the cosmogony. If, for example, the spiral is the form normally assumed by a contracting nebulous mass, the idea at once suggests itself that the solar system has been evolved from a spiral nebula, while the photographs show that the spiral nebula is not, as a rule, characterized by the simplicity attributed to the contracting mass in the nebular hypothesis. This is a question which has already been taken up by Professor Chamberlin and Mr. Moulton, of the University of Chicago.

The Crossley reflector promises to be useful in a number of fields which are fairly well defined. It is clearly unsuitable for photographing the Moon and planets, and for star charting. On the other hand, it has proved to be of value for finding and photographically observing asteroids whose positions are already approximately known.

One of the most fruitful fields for this instrument is undoubtedly stellar spectroscopy. Little has been done in this field as yet, with the Crossley reflector, but two spectrographs, with which systematic investigations will be made, have nearly been completed by the Observatory instrument maker. One of these, constructed with the aid of a fund given by the late Miss C. W. Bruce, has a train of three 60° prisms and one 30° prism, and an aperture of two inches; the other, which has a single quartz prism, will, I have reason to expect, give measurable, though small, spectra of stars nearly at the limit of vision of the telescope.

The photogravure of the Trifid nebula which accompanies this article (see frontispiece) was made from a photograph taken with the Crossley reflector on July 6, 1899, with an exposure of three hours. It was not selected as a specimen of the work of

the instrument, for the negative was made in the early stages of the experiments that I have described and the star images are not good, but rather on account of the interest of the subject. At the time the photogravures were ordered no large scale photograph of the Trifid nebula had, so far as I am aware, ever been published.[1] The remarkable branching structure of the nebula is fairly well shown in the photogravure, though less distinctly than in the transparency from which it was made. The enlargement, as compared with the original negative, is 2.9 diameters, (1 mm = 13″). The fainter parts of the nebula would be shown more satisfactorily by a longer exposure.

[1] Since then a photograph by Dr. Roberts has appeared in *Knowledge*, **23**, 35, Feb. 1900.

THE PHYSICAL MEANING OF THE STAR-MAGNITUDE.

By R. DE KÖVESLIGETHY.

By assuming Pogson's numerical value for the constant of the Fechner psycho-physical law, we get the definition of the star-magnitude in the form

$$m - m_0 = -2.5 \log \frac{J}{J_0} \qquad (1)$$

which, for stars equal to or fainter than the sixth magnitude, represents tolerably well the connection between the measured intensity J and J_0 and the estimated magnitude m, m_0 of any two stars. This definition, however, involves merely subjective physiological elements, inasmuch as the visual intensity depends not only upon the objective continuous spectrum of the star, but varies also with the limiting wave-lengths of perception and with the sensibility of the eye, which is itself a function of the wave-length. Thus even for another suitable choice of the Pogson constant the differences of magnitude seem altered when the perceiving medium of the spectrum is changed; *e. g.*, when the photographic film is used instead of the normal eye.

The question of a physical definition of the star magnitude is therefore still open, though in many cases it might be of considerable value, especially for the estimation of processes operating in new stars. Of course I do not here allude to the fact sometimes invoked in investigations of the dimensions of the stellar system, that stars of equal intensity seem fainter by *one* magnitude if removed to a 1.585-fold distance, but I seek the changes in the physical condition of a star whose brightness has varied by *one* magnitude.

Let

$$i = \Lambda f(\lambda, \mu) \qquad (2)$$

be the intensity of a continuous spectrum between the wave-lengths λ and $\lambda + d\lambda$. Then the visual intensity becomes

$$I = \Lambda \int_{\lambda_1}^{\lambda_2} s f(\lambda, \mu)\, d\lambda, \qquad (3)$$

where the sensibility of the eye, s, is a function of the wave-length, while λ_1 and λ_2 denote the limits of the visual spectrum. The function $\Lambda f(\lambda, \mu)$ contains at least two independent parameters, as I have shown elsewhere[1] (since it would lead to an equation of dispersion without any constant). Let Λ be the total intensity of the spectrum and μ the wave-length of the maximum of intensity. Thus we have

$$\int_0^\infty f(\lambda, \mu)\, d\lambda = i .$$

The analytical form of the sensibility function s is quite unknown (for geometrical representation, see this JOURNAL, **11**, 18–22). Numerically it might be determined by comparisons of the visual intensity with energy observations made with an absolutely black bolometer. Its differential equations could be measured in a simpler way by the probable error with which a homogeneous light-field of given wave-length can be inserted on the appropriate place of a continuous spectrum. Here we can get on without the explicit expression of the sensibility, inasmuch as a mean value of it between the limits of the spectrum may be placed as a factor before the integral. Thus the visual intensity becomes

$$I = \sigma \Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda , \qquad (4)$$

and σ remains constant for all stars of the same spectral composition, that is of the same type, while it varies from type to type.

This variation is very little, however, and, beyond the limits of exactness of photometric measures, may be neglected in practice. Indeed all photometric measures, even of differently colored stars, are usually made by comparison with a few standard stars, and likewise the same color of comparison star was always used in the *Potsdamer Durchmusterung*, being about equal to the mean color of the second stellar type.

[1] *Monthly Notices*, **58**, No. 3, 115; *Grundzüge einer theoretischen Spektralanalyse* Halle a/S. 1890, p. 151.

Thus we become—at least in practice—independent of the unknown coefficient σ, and the Pogson formula gives for the same star whose magnitude changed by *one* order $(m=m_0+1)$:

$$1 = -2.5 \log \frac{\Lambda \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu)\, d\lambda}{\Lambda_0 \int_{\lambda_1}^{\lambda_2} f(\lambda, \mu_0)\, d\lambda}, \tag{5}$$

as conditional equation for the changes necessary in Λ_0 and μ_0 to produce such effect. Taking into consideration what we have said of the sensibility factor, this equation holds good, even if the change of light was accompanied by a variation of the color amounting to an interval of two types.

The two parameters of the spectrum, Λ and μ, are functions of the temperature and the density,[1] and, though known functions, do not allow a uniform solution of the above equation. The manner in which the temperature and density changed remains in every case quite arbitrary, and thus a uniform physical definition of the star magnitude cannot be given.

The question becomes resolvable in a uniform manner, however, if we assume that the radiating body is an absolutely black one. Inasmuch as the superficial layer, which produces the greatest part of the continuous spectrum, may be regarded thick enough, our hypothesis holds also good for the fixed stars.

Now we know two analytical expressions for the continuous spectrum; the one generally applicable for any body is by the author:[2]

$$i = \frac{4}{\pi} \mu \Lambda \frac{\lambda^5}{(\lambda^2 + \mu^2)^3}\ ; \tag{6}$$

the other, which refers only to absolutely black bodies, is that of Wien-Paschen,[3] and may be written in the form

$$i = \frac{5^4 \mu^4}{6} \Lambda \lambda^{-5} e^{-\frac{5\mu}{\lambda}}. \tag{6'}$$

[1] "Ueber die beiden Parametergleichungen der Spektralanalyse," *Math. u. Naturw. Ber. aus Ungarn*, **16**, 1–49.

[2] *Grundzüge e. theor. Spektralanalyse*, p. 157.

[3] This JOURNAL, **10**, 40.

For an absolutely black body, Λ obeys the Stefan law of radiation, while μ is given by the Wien formula. If θ denotes the absolute temperature, then

$$\frac{\Lambda}{\Lambda_0} = \frac{\theta^4}{\theta_0^4}, \text{ and } \mu\theta = 2880, \tag{7}$$

provided that μ is given in thousandth parts of the millimeter. It may be said, in brief, that I was familiar with this ten years before the publication of Wien's *Verschiebungsgesetz*, and that I determined its constant provisionally by the observations of G. Müller[1] on the absorption of the air, in a most troublesome way, as 2088. Similarly I used this equation and the value of the constant for the determination of the temperature of the Sun's chromosphere. It gave 1800°, or, with the present value of the constant, 2475° of the absolute scale, being 1.1631 the wavelength of the maximum intensity of an absolutely black body having the same temperature as the chromosphere.[2]

Putting now

$$F_1(\mu) = \frac{4}{\pi}\mu \int_{\lambda_1}^{\lambda_2} \frac{\lambda^2}{(\lambda^2+\mu^2)^2} d\lambda$$

$$= \frac{2}{\pi}\left[\text{arc tang } \frac{\mu(\lambda_2-\lambda_1)}{\mu^2+\lambda_1\lambda_2} - \mu\,\frac{(\lambda_2-\lambda_1)(\mu^2-\lambda_1\lambda_2)}{(\lambda_1^2+\mu^2)(\lambda_2^2+\mu^2)}\right], \tag{8}$$

and

$$\frac{\lambda_1}{\mu} = \text{tang } \phi_1\,; \quad \frac{\lambda_2}{\mu} = \text{tang } \phi_2,$$

we get in a more simple form

$$F_1(\mu) = \frac{2}{\pi}\left[(\phi_2-\phi_1) - \sin(\phi_2-\phi_1)\cos(\phi_2+\phi_1)\right],$$

and similarly in the other equation

$$F_2(\mu) = \frac{5^4\mu^4}{6}\int_{\lambda_1}^{\lambda_2} \lambda^{-5} e^{-\frac{5\mu}{\lambda}} d\lambda$$

$$= \frac{125}{6}\left\{ e^{-\frac{5\mu}{\lambda_2}}\left(\frac{\mu^3}{\lambda_2^3} + \frac{3\mu^2}{5\lambda_2^2} + \frac{6\mu}{25\lambda_2} + \frac{6}{125}\right)\right.$$

$$\left. - e^{-\frac{5\mu}{\lambda_1}}\left(\frac{\mu^3}{\lambda_1^3} + \frac{3\mu^2}{5\lambda_1^2} + \frac{6\mu}{25\lambda_1} + \frac{6}{125}\right)\right\}, \tag{8'}$$

[1] *Astr. Nachr.*, 103, 241, 1882.

[2] *Grundzüge*, p. 185.

we have generally

$$I = \Lambda F(\mu) , \tag{9}$$

and thus the Pogson equation (5) becomes

$$\log \mu - \frac{1}{4} \log F(\mu) = 0.1 + \log \mu_0 - \frac{1}{4} \log F(\mu_0) . \tag{10}$$

Thus if we know the average values of μ_0 for the individual stellar types we may calculate the variations in μ necessary to produce a change from *one* magnitude in the luminosity of the star. Early spectral-photometric observations of thirty-four fixed stars[1] led me to the following values, corrected for the effect of the atmosphere:

Type of the star:		I.	II.	III.	
Average μ_0	=	0.45	0.53	0.60,	(11)

which give, according to the two hypotheses made for $F_1(\mu)$ and $F_2(\mu)$, the limits of the visual spectrum being assumed at $\lambda_1 = 0.39$ and $\lambda_2 = 0.76$, the equations

I. type;	$\log \mu - \frac{1}{4} \log F_1(\mu) + 0.0923 = 0$	and	$\log \mu - \frac{1}{4} \log F_2(\mu) + 0.1679 = 0$			
II.	" "	$+0.0082 = 0$	"	"	$+0.0881 = 0$	(12)
III.	" "	$-0.0592 = 0$	"	"	$+0.0200 = 0$,	

The solutions for the three types are, accordingly,

I. type	$\mu = 0.546$	and	$\mu = 0.551$
II.	0.636		0.633
III.	0.715		0.706,

(13)

where the first values refer to the first hypothesis for $F_1(\mu)$, the other column to the second hypothesis $F_2(\mu)$.

For facilitating the solution of (10) we give here a short numerical table of the function $F_1(\mu)$ and $F_2(\mu)$.

μ	Log $F_1(\mu)$	Log $F_2(\mu)$	μ	Log $F_1(\mu)$	Log $F_2(\mu)$
0.40	9.4073	9.6827	0.65	9.2354	9.5412
45	3821	6842	70	1931	4818
50	3507	6671	75	1499	4162
55	3150	6357	80	9.1066	9.3454
60	9.2762	9.5930			

The part of the energy falling in the visual spectrum is considerably greater if the spectrum is assumed in the exponential form instead of the algebraic form. But in both cases the

[1] *Beobacht. angest. am astrophys. Observ. O-Gyalla*, 9, 21–41, 1888.

intensity vanishes for $\mu = 0$ and $\mu = \infty$, and reaches a maximum value for $\mu = 0.309$ and $\mu = 0.429$ respectively. These values, therefore, correspond to the impression of the purest white in the mixed light of the source. I remark that the gradual increasing and the subsequent decreasing of the intensity with increasing temperature is proved, at least in single cases, by observations of Lucas.

In virtue of the second part of equation (7), the solutions (13) correspond to the following temperatures:

Type.	θ_0	θ_1	θ_2	$\frac{\theta_0}{\theta_1}$	$\frac{\theta_0}{\theta_2}$	
I.	6400°	5274°	5230°	1.213	1.224	
II.	5434	4527	4547	1.200	1.195	(14)
III.	4799	4025	4080	1.191	1.176	
			Mean of temperature ratio:	1.201	1.198.	

Therefore, if a white star changes in magnitude by *one* order, it is the same as if its superficial temperature fell from 6400° to 5274° or to 5230° respectively. The temperatures in the two hypotheses are almost identical, and the ratio of them is also sensibly the same, but shows a slight variation from type to type. If we therefore assume on the two hypotheses that a variation of *one* magnitude corresponds to an increase of temperature in the ratio of 1:1.201 or of 1:198, the equation (1) may be written in this form:

$$m - m_0 = -12.58 \log \frac{\theta}{\theta_0}, \text{and } m - m_0 = -12.74 \log \frac{\theta}{\theta_0}, \quad (15)$$

and the assumption of an average ratio gives for the stars of the first and third type errors of 0.056 and 0.044, of 0.118 and 0.103 mag., respectively, which on my hypothesis do not reach, on the other scarcely exceed, the limits of exactness of photometric measures.

We have, then, the following theorem:

When the light of a star in form of a point, of nearly absolute black character, increases by one magnitude, then, independently of the color of its light, its superficial temperature has lessened by 20 per cent.

We may further remark, that the "superficial" temperatures (*i. e.*, temperature of the layer, which produces the principal part of the continuous spectrum) given in (14) under θ_0 represent lower limits. For if any body and an absolutely black body have the same wave-length of intensity maximum, the latter is always of lower temperature.

The above investigation may be of some use in its application to new stars. The star *T Coronae* increased by three magnitudes in about two and one half hours; that is to say, during this time its temperature was elevated about 0°.5 per second, a most probable quantity, whether we assign the apparition to a collision with a cosmic cloud or to an eruption of the still fluid inner magma.

O-GYALLA OBSERVATORY,
February 1900.

ON THE ESCAPE OF GASES FROM PLANETARY ATMOSPHERES ACCORDING TO THE KINETIC THEORY.

By G. JOHNSTONE STONEY.

PART II. MORE DETAILED EXAMINATION.

MR. COOK in America[1] and Professor Bryan in England[2] have endeavored to determine the rate of outflow of gaseous molecules from atmospheres, by regarding as applicable to that part of the atmosphere from which the outflow takes place, probability laws which refer to the distribution of the speeds of the molecules in a portion of uniform gas surrounded by similar gas. In Part I of the present paper some general considerations have been adduced tending to show that this use of the aforesaid laws is not legitimate; and in the following pages it is proposed to institute a closer scrutiny, of such a kind as will bring the assumptions involved fully into view, and exhibit in a clear light the contrast between them and what is really going on in nature. In order to do this it will be convenient to inquire successively into the molecular velocities of the lower and of the upper regions of the atmosphere.

MOLECULAR MOVEMENTS IN THE LOWER STRATA OF THE ATMOSPHERE.

1. Maxwell's law and its successors the Boltzmann-Maxwell law, and the law with the additional term introduced by Professor Bryan, are probability laws, and as such depend essentially on the solution of a functional equation. This method of proof does not ascertain that a law of the kind exists, but only that if such a law had existed under the conditions assumed in the proof, it would have been that law which emerges when we solve the functional equation. The question as to whether a law of

[1] See the January number of this JOURNAL.

[2] See the *British Association Reports* for 1893 and 1899; and the *Proceedings of the Royal Society* for April 1900.

the kind exists is one which the proof of the probability law gives no answer. It is there assumed.

2. This is consistent with what we know of such laws, that *the law is not really complied with* for any finite value whatever of N, using N for the number of events whose distribution is under consideration. But the probability law is nevertheless of value whenever it can be shown that for values of N above a certain limit N_0, the actual distribution is unlikely to differ much from that laid down by the law. We can assure ourselves that this is the case in many instances. We find, however, that the number N_0—the smallest number of events whose distribution will exhibit a sufficient approach to the law to be of use—has very different values in different problems. In some it is a moderate number; while in others (including, as we shall see, the case of molecular speeds) it is an enormously large number.

3. In some few instances the value of N_0 may be foreseen; as when the law concerns the throwing of dice. Here the probability of each individual event is known, the causes of the events are alike with respect to the chance as to which of certain definite possible events shall follow, and we have all the data required to compute the value of N_0 which suffices to render probable any desired degree of approximation between the actual distribution and that furnished by the probability law.

But in most cases we have to find N_0 by experiment. Thus, if the matter to be investigated is the distribution of shots on a target, we must have recourse to experiment to determine what number N_0 of shots will have to be fired under given conditions in order that it may be probable that the actual distribution shall sufficiently accord with that indicated by the law; and if we vary the conditions the number N_0 is found to vary.

4. In both these cases the individual events are independent of one another, and their causes are alike. Where this is so, the number N_0 is usually not a very large number. But when the problem has reference to the distribution of the speeds with which gaseous molecules pursue their free paths, the individual events neither are independent nor are their causes alike with

regard to the probability of the consequences that will ensue. Each speed is the outcome of an intricate web of preceding speeds and of associated events which are all the more complex because of exchanges which take place during the encounters between the kinetic energy of the molecules regarded as missiles and the energy of those wholly different internal events which influence and are influenced by heat radiations and other kinds of electrical activity. The many causes which have thus to combine to develop one of the molecular speeds may, and frequently do, coöperate to produce in it an exaggerated effect. *There is, accordingly, a continual recrudescence of abnormal speeds always springing up within a gas.*

5. The frequency of combinations that produce exaggerated effects is consequent upon the marvelous rapidity and complex variety with which encounters succeed one another in the career of every molecule. Thus the number of encounters met with by each molecule of the air about us at the bottom of the atmosphere is some seven or eight millions in every thousandth of a second of time, according to Maxwell's determinations. It follows from this that a combination of events as unusual as the throwing of doublets thirteen times in succession upon unloaded dice, the probability of which is less than 1 in 13,000 millions, does on the average determine the speed of *every* molecule of the air about us rather more frequently than once every two seconds.

A most instructive experiment which places in a clear light (*a*) the frequent occurrence of exceptional events in gases, (*b*) the fact that they may reach every molecule, and (*c*) the conspicuousness of their effects, may be made by exposing one mixture of equal volumes of hydrogen and chlorine to diffused light and another to direct sunshine. In diffused light the gases slowly combine into hydrochloric acid gas, in other words, the exchange of chemical atoms between two encountering molecules is, in diffused light, consequent upon some very unusual kind of encounter which occurs excessively rarely from the molecular point of view. In intense light this kind of

encounter is still infrequent from the molecular standpoint, but so much less so than before that the combination of the gases now seems to our coarse human senses to take place with explosive rapidity. In either case, after the lapse of a sufficient time the reaction is complete; every molecule is now hydrochloric acid, *i. e.*, the very unusual kind of encounter has reached *every* molecule of the hydrogen and chlorine which was originally present.

6. It thus appears that the speed of a molecule in one of its free paths is the outcome of an intricate body of causes, some of which are preceding molecular speeds in the gas, and others are agencies of a different character; and further that these various factors sometimes find themselves so interwoven that they produce exaggerated effects. When in this way an abnormal speed has been developed in a molecule, the subsequent encounters of that molecule—which at the bottom of the atmosphere no molecule is able to avoid—tend in almost every case to tone down the irregularity. Now this toning down must have been effectual upon a sufficient majority of the irregularities that present themselves, in order that the distribution of the speeds may approximately conform to such a law as Maxwell's. This requires a very large number of molecular speeds to be embraced in our survey; so that in gases, N_0 is likely to be a vastly higher number than in the simple cases with which we are more familiar, such as those referred to in paragraph (3). This inference is borne out by experiment.

7. Let ΔV be a small volume of air under standard temperature and pressure, containing n molecules and surrounded by air subjected to the same conditions; and let Δt be a small duration such that in it each molecule on the average meets with n' encounters: then will

$$N = n \,.\, n'$$

be the number of free-paths described in the time Δt, by the molecules which were, at the beginning of that time, within the volume ΔV.

We have to find by experiment the value of N_0, *i. e.*, the smallest value which N can have consistently with a recognizable

approximation between the actual distribution of speeds, and that indicated by the probability law. Now several experiments, both in ordinary air and in attenuated air, seem to show that the limiting number N_0 is approached in ordinary dry air (which consists mainly of nitrogen, oxygen and argon)

when ΔV is the cube of 0.1 of a millimeter,
and Δt is the $\frac{1}{2^{15}}$ of a second of time.

We have next to turn these into the corresponding number of free paths. The estimates and determinations of the number of molecules present in a gas under standard temperature and pressure, differ a good deal from one another, but may all be summed up in the statement that

> In a cubic centimeter of gas under standard temperature and pressure there are —
>
> *several* trillions of molecules;

where we use the word "several" in a special and very convenient sense to mean a number which differs in the various determinations and the precise value of which is, therefore, not known, but which certainly stands somewhere between the limits 10 and 1000.

For our present purpose we shall find it convenient to extend this statement to the air around us at the bottom of the atmosphere, whatever the indications of the thermometer and barometer for the time being may be. This can be effected by slightly enlarging the range within which the number represented by "several" is to be sought. When thus modified the statement becomes

> In air or gas, subjected to any of the temperatures and pressures we have experience of at the bottom of the atmosphere, there are in each cubic centimeter—
>
> *several* trillions of molecules;

where "several" stands for some number between 8 and 1100.

Hence in ΔV (ΔV being the cube of the tenth of a millimeter) the number of molecules of ordinary air is

n = *several* billions.

Again, as each molecule in atmospheric air under, or nearly under, standard conditions, meets with about 7000 or 8000 millions of encounters per second (see Maxwell's paper), it follows that in the time Δt ($\frac{1}{2}\frac{1}{5}$ of a second) each molecule of air at the bottom of the atmosphere meets with about

$$n' = 300 \text{ millions of encounters.}$$

Hence the number of free paths in the time Δt, of the molecules that at the commencement of that time were within the volume ΔV, is about—

$$N_0 = n \,.\, n' = \textit{several} \text{ times } 300 \text{ trillions,}$$

where "several" means a number between 8 and 1100, the value of which is not more exactly known.

8. This then seems to bring us into the neighborhood of the number of individual molecular speeds which must be included in our survey, in order that the distribution of these speeds at the bottom of our atmosphere, and in the $\frac{1}{2}\frac{1}{5}$ of a second, may approximate to obeying a probability law such as the Maxwell law, the Boltzmann-Maxwell law, or the Bryan-Boltzmann-Maxwell law. No doubt, if we assign a different value to Δt, we may get a somewhat different value for N_0; but it will always be an immense number. This is because the assumption that is made in the proof of Maxwell's law — that the proportion of molecules whose speed lies between v and $v + dv$, is dv multiplied by a function of v only — is too great a simplification of the far more complex function which it really is.[1]

9. The state of affairs at the bottom of the atmosphere will perhaps be more fully apprehended, if we represent it to ourselves under a symbolical form.

Out of N free paths, the actual number whose speed lies between v and $v + dv$

$$= N(\pi + \delta)\, dv$$

where π is the probability function — Maxwell's law or one of

[1] See the first few pages of a paper on "The Kinetic Theory of Gas," by the present writer, in the *Proceedings of the Royal Dublin Society* for June 26, 1895, or in the *Philosophical Magazine* for October 1895.

the others — and δ is the function which represents the deviation of the actual from the computed distribution.

Now π is assumed by Maxwell to be a function of v only, and similar assumptions are made with reference to the other laws ; from which it follows that δ must be a very complex function of many variables — v, n, n', θ, τ, etc., where v is the speed, n the number of molecules in ΔV, n' the number of encounters which each molecule meets with in the time Δt, θ the average duration of a free path, τ the average duration of an encounter, and where etc. stands for other variables depending on whether the encounters are between molecules of the same or of different gases, and on the events which happen during encounters — which in turn depend on the kind of gas or gases, and on the electro-magnetic waves or other electrical agencies operating through the ether. It is essential that those variables shall be included which when suitably combined will furnish its function δ whatever is the proper mathematical expression of the *cumulative* effect which arises when opportunity offers. Cumulative effects are partly a consequence of the influence which the velocities exercise over all their successors, and partly due to the varying foreign agencies which also intervene. Whenever the many factors more or less coalesce, then there emerges a more or less cumulative effect. Now we can conceive this to be expressed mathematically in function δ, although nothing that any mathematician could write down would at all adequately represent it. In such cases we must have recourse to experiment or observation to discover what, at any given time, is the actual outcome of function δ. Moreover, this function δ is not fixed : it fluctuates, and has had a very different value at some times from what it has had at others. In the present state of the Earth it is larger in sunshine than in shade ; perhaps markedly different during a thunderstorm or when there is an auroral display from what it is at other times ; probably sensitive to all the influences which cause upheavals of molecules into stratum V[1] ; and so on.

[1] See paragraph 12, p. 364

When the events whose distribution is being studied are so much alike as they are in the problems spoken of in paragraph (3), function δ is little more than a mere function of V and N, and one which dwindles away in value according as N increases; but where the events whose distribution is sought are due to complex, differing, and fluctuating causes, function δ may hold a much more conspicuous position in its relation to function π, and one which is likely to become predominant for certain values of v, especially at certain times.

10. The general lesson which we learn from this way of presenting the subject is that probability laws, such as Maxwell's, when applied to the distribution of molecular speeds, may be useful guides in reference to those values of v, which correspond to large values of π; but should not be trusted, even in the lower strata of the atmosphere, in reference to the values of v which assign small values to π. Now it is only with these very values of v that we are concerned when studying the possibility of the escape of gas from an atmosphere.

11. At the bottom of the atmosphere, where δV is a volume of air surrounded by air at almost exactly the same temperature and pressure, there is practically no chance that an irregularly moving molecule can avoid that torrent of subsequent encounters which is what in the air about us tones down and as it were swamps all irregularities, and brings about such an approach towards a *general* conformity with Maxwell's law as is possible; but the essentially different environment in which a molecule finds itself in the upper reaches of the atmosphere, provide it with new opportunities, to the study of which we have now to invite attention.

MOLECULAR MOVEMENTS IN THE UPPER STRATA OF THE ATMOSPHERE.

12. *Definitions.* — It is convenient to picture the atmosphere as consisting of 26 strata distinguished from one another by the letters of the alphabet, A being the stratum at the bottom in which we live and Z being the stratum at the top.

When we describe a molecule which has left the Earth as having escaped from a specified stratum, what is to be understood is that it was within that stratum that the molecule met with its last gaseous encounter, whereupon it ceased to be a molecule of gas, and started on its new career as a body traveling by itself through space.

13. Now it is manifest that the escape of gas from an atmosphere can only take effect from its upper regions, since a molecule situated low down has no appreciable chance of making its way outwards without meeting with encounters on the way. In fact it will presently appear that we can definitely locate the escape as coming exclusively from three strata which we shall call the ultimate stratum of the atmosphere *Z;* next to it the penultimate stratum *Y;* and farther down the ante-penultimate stratum *X*. Beneath these come the other strata of the atmosphere, from *A* to *W*, from none of which has a gaseous molecule any prospect of being able to make its way outwards without encountering some other molecule.

14. *Definitions of these three strata.*—Stratum *Z*, at the boundary of the atmosphere, *is characterized by an almost total absence of gaseous encounters* within it. The molecules that tenant it have been thrown up from below, and are so separate that they scarcely ever come across one another while within that stratum. Most of them describe elliptic trajectories relatively to the Earth, and fall back into the penultimate stratum: if any have risen into it in hyperbolic trajectories these leave the Earth altogether: so may some that rise into it in elliptic trajectories if they are sufficiently long to carry the molecule within the perturbating influences of the Sun or Moon.

Next under stratum *Z* comes the penultimate stratum *Y* throughout which encounters do take place, and which is characterized by the circumstance that *the molecules which occupy it are all within effectual striking distance of stratum Z* and of the space beyond. By the molecule's being within effectual striking distance of the ultimate stratum and of the void beyond, is meant that if a molecule after an encounter anywhere within stratum *Y*,

finds itself traveling with suitable velocity as regards speed and direction, it has, in the upper ranges of stratum Y a great, in its middle region a moderate, and in its lowest portion an appreciable chance of escaping outwards *without meeting with any further encounter* within stratum Y.

The rest of that part of the atmosphere from which molecules sometimes escape lies farther down than the penultimate stratum, and may be called the ante-penultimate stratum X. Molecules after an encounter within this stratum, whatever their speed and direction, have but a slender prospect of avoiding further encounters; but *as this does sometimes happen*, though not often, stratum X must be included among those from which the escape of gas is possible.

15. These three strata are all of great depth, by reason of the attenuation of the air at great altitudes: they also obviously pass insensibly into one another, but may nevertheless be as distinct in the atmosphere as are the chin, the cheek, the temple and the forehead of the human countenance, which like them have no definite lines of demarcation.

16. Now it is apparent from the definitions of these strata, that *nearly the whole of the actual escape* of gas from the Earth consists of molecules which met with their last encounter within the penultimate stratum. An altogether trifling proportion have emanated from the adjoining strata above and below — few from the ultimate stratum because there are few encounters there, and few from the ante-penultimate stratum because few molecules that have encountered within this stratum can escape a subsequent encounter; and practically none come directly from any of the strata that lie farther down. Accordingly what we have to do is to consider what velocities may arise within the penultimate stratum. In doing this we shall have to bear in mind that a molecule within that stratum moving swiftly outward is traveling toward regions where there are fewer and fewer encounters, so much so that it may get off without meeting with any encounter; while a molecule within the stratum that is traveling swiftly inward is plunging into regions where encounters are inevitable.

17. It would be rash to guess how many miles deep this penultimate stratum is, though we can see that it is of great depth. And if we make Δt 4 or 5 seconds, which is a reasonable value to assign to it in view of the time which a molecule with the requisite speed would take to escape, the corresponding ΔV, in order to provide a sufficient number of free paths during the time Δt, of the molecules that at the beginning of that time were within the space ΔV, may on a probable hypothesis be a cube of some 10 or 20 kilometers, *i. e.*, a cube of from 6¼ to 12½ miles. This then may be taken as about the smallest volume within which any law can be recognized governing the distribution of the speeds that occur within it, in periods of time as short as Δt. And that law, if we could discover it, would doubtless be *utterly unlike Maxwell's law* or either of its successors. It would be a resultant of distinct laws belonging to the different directions, in which the behavior of molecules traveling outward is discriminated from the behavior of molecules traveling inward, and in which a due allowance has been made for the circumstance that the molecules of the lighter gases whenever a sufficient speed is engendered in them, are likely, if the direction of their flight is outward, to place themselves beyond the reach of all subsequent encounters; thus destroying in their case the machinery which lower down in the atmosphere brings about a partial conformity to such a law as Maxwell's.

18. Another and very important factor which would need to be taken into account in determining the distribution of speeds at great elevations in the atmosphere, is that the molecules in those regions are exposed to the full glare of a sunshine which is not moderated by having passed through the Earth's atmosphere, and that whatever internal activity is thereby excited within the molecules during their comparatively long flights, will on their next encounter contribute energy towards the speed with which the two colliding molecules will fling asunder. How much energy may be introduced in this way is revealed to us by the intensely dark terrestrial lines of oxygen in the solar

spectrum, and by the very remarkable fact that all radiations from the Sun of wave-lengths less than 0.3 of a micron, are completely taken up and intercepted by the upper portions of our atmosphere, so that none of them reach its lower strata. It is also brought home to us by the hydrochloric experiment cited above in paragraph 5, where unusual events become much more numerous in direct sunshine.

19. In view of all these facts the present writer came to the conclusion more than thirty years ago (*a*) that *it is a mistake* to suppose that *Maxwell's* law governs the distribution of speeds in that upper portion of the atmosphere from which alone molecules escape ; and (*b*) that the true law of distribution, whatever it is, has but a partial connection with the rate of outflow (1) on account of the size which has to be attributed to ΔV, (2) on account of the very different behavior of outflowing and inflowing molecules, (3) on account of the magnitude and fluctuating character of the deviation function δ, described in paragraph 9, and (4) on account of the opportunity which happens now and then to each molecule of the lighter gases, of withdrawing from those subsequent encounters which would keep it within the jurisdiction of the law.

20. My first attempt was to try whether the rate of escape can be determined by the help of a law of the same kind as Maxwell's, in which the vectors as well as the scalar values of the velocities were taken into account. But this, too, proving to be insufficient, I turned to the *a posteriori* method of investigation, and sought to gain some insight into what the outflow has actually been, by studying its effects. The observations principally made use of are

1. That the Moon is now without an atmosphere ;
2. That the Earth has been able to retain the vapor of water in its atmosphere ;

both of which are data that may be depended upon.

21. It would be very satisfactory if we could also investigate the problem in the more *a priori* way that has been attempted by Messrs. Cook and Bryan ; but for the reasons

stated above this seems to the present writer to be quite beyond the reach of the Molecular Physics we as yet know.

APPENDIX.

ON THE BEHAVIOR OF HELIUM IN THE EARTH'S ATMOSPHERE.

The questions whether helium in former times escaped from the Earth, and whether it is still escaping, have a double interest—they are of interest in themselves, and of interest because they seem to answer the further question whether we are to attribute the polar snows of *Mars* to water or to carbon dioxide.

The attempts hitherto made to obtain *a priori* answers to questions of this kind from the kinetic theory of gas, have, as appears from the discussion of them in the preceding pages, afforded no real light. They have in fact had to be deductions from illegitimate premises, based upon the assumption that the events as they occur in nature are far simpler than they really are. Nor does a valid investigation upon these lines seem even possible; the knowledge we as yet possess of molecular physics is too fragmentary and defective to deal adequately with the problem, and our acquaintance with the penultimate stratum of the atmosphere, which is the situation from which almost the whole of the real escape takes place, is imperfect.

It becomes then of importance to proceed by the other possible method, the *a posteriori* method, which rests upon such data as are furnished by experiments, *i. e.*, by observation of the actual conditions under which helium exists upon the Earth. The facts brought to our knowledge in this way appear to indicate that helium is still slowly escaping from the Earth, and *a fortiori* was able to escape more freely in earlier cosmical ages. The data upon which this conclusion rests are now fuller and more definitely known than they were when the present writer ventured, some years ago, to put forward the view as probable. They seem to change what was then probability into what is now almost certainty. These more precise data have been generously furnished to me by my friend, Professor Ramsay, and are as follows:

1. The proportion by volume of argon in dry atmospheric air is about 1 per cent. of the whole; the volume of neon (to which the present notes will not further refer) may be taken as about a thousandth part of the volume of argon, and the volume of helium as about $\frac{1}{10}$ to $\frac{1}{20}$ of the volume of neon. Accordingly the volume of helium in dry air is something like from $\frac{1}{10000}$ to $\frac{1}{200000}$ of the volume of argon.

2. Both argon and helium are supplied to the atmosphere by hot springs; argon generally by all hot springs which contain atmospheric gases, and helium by some of them.

3. The argon in such springs, like the oxygen and nitrogen, may be simply gas which had previously been removed from the atmosphere by water. A liter of water under ordinary conditions will absorb:

about 45 cc of the oxygen of the air in contact with it,
about 15 cc of its nitrogen,
about 40 cc of its argon,[1]
about 14 cc of its helium.[1]

Hence in rain we should expect to find the following proportions preserved:

$$\frac{20.9}{100} \times 4.5 \text{ of } O_2$$

$$\frac{78.1}{100} \times 1.5 \text{ of } N_2$$

$$\frac{1}{100} \times 4 \text{ of } A$$

$$\left.\begin{array}{ll} \text{and from} & \dfrac{1}{1{,}000{,}000} \times 1.4 \\ \text{to} & \dfrac{1}{2{,}000{,}000} \times 1.4 \end{array}\right\} \text{ of } He$$

So far as oxygen, nitrogen, and argon are concerned, these proportions are sufficiently nearly those in which the gases are present in the springs referred to. But in those springs in which

[1] For the determinations made by Herr Estreicher in Professor Ramsay's laboratory, see *Zeitschrift für physikalische Chemie*, 31, 148, 1899.

helium also has been detected it seems to be present in quantities about $\frac{1}{10}$ of the argon — that is, in a quantity which is nearly from 3000 to 6000 times more than is consistent with its having been derived from the atmosphere.

4. This great excess of helium in some springs has doubtless a mineral origin, some minerals, chiefly uranium compounds, containing much helium, which they give up when heated. On the other hand there does not appear to be any comparable mineral source of argon.

5. Hence, on the whole, the argon which is being supplied to the atmosphere by hot springs seems to be argon which had previously been withdrawn from the atmosphere, and which is being restored to it. Whereas, in contrast to this, there seems to be a continuous transfer of additional helium from the solid earth to the atmosphere always going on.

Thus the facts seem to indicate with some emphasis:

1. That the excessively small quantity of helium in the atmosphere is helium on its way outward.

2. That it would have become a larger constituent of the atmosphere by reason of the influx from below if there had been no simultaneous outflow from above.

3. That the rate of this outflow is presumably equal to the rate of supply; and therefore such as would suffice, in a few thousand,[1] or at least in a few million years, to drain away the

[1] The total quantity of helium in the Earth's atmosphere, according to Professor Ramsay's determination of its percentage in air at the bottom of the atmosphere, would, if reduced to standard temperature and pressure, nearly occupy something between a cube of ten miles and a volume half that size. The actual volume is, in fact, a little less than this, inasmuch as the computation has been made without taking into account that part of the reduction of density of the helium as it ascends through the atmosphere, which is caused by its being able to escape from the top.

Now, so far as can be judged from the observations upon such hot springs as have been examined, it seems likely that the rate at which helium is at present being filtered from the Earth into the atmosphere is such as would fill a cube of ten miles with helium at standard temperature and pressure in something like one or two thousand years, or perhaps in a less time. Hence the time which the helium at present in the atmosphere would take to escape if the supply from below were cut off, may be taken to be a period of this order of magnitude.

stock of helium in the atmosphere if the source of supply from below could be cut off.

Hence the evidence goes to show (1) that argon is not able to escape from the Earth; (2) that helium is escaping from the Earth, and consequently that water can probably escape from *Mars*, and that the polar snows of that planet are presumably carbon dioxide.

LONDON, April 1900.

ON THE PROGRESS MADE IN THE LAST DECADE IN THE DETERMINATION OF STELLAR MOTIONS IN THE LINE OF SIGHT.

By H. C. VOGEL.

AFTER the early attempts at the determination of the component in the line of sight of the motion of the stars by means of the spectroscope, which were made in 1868 by Huggins, in London, and in 1871 by myself at Bothkamp, on a few stars, extensive observations of this kind were conducted at the Observatory at Greenwich, extending over a period of thirteen years. The great persistence exhibited by Maunder in these observations, which were placed in his charge, is worthy of the more recognition in view of the slight interest which astronomers then generally had in the physical side of astronomy, and especially in view of their skeptical attitude toward the application of the spectroscope for determinations of motion. There was, indeed, some basis for this, inasmuch as a contention had arisen among the physicists as to whether the so-called Doppler's principle, which was recognized as experimentally correct for sound waves, and which also permitted an easy theoretical explanation, could be justifiably transferred directly to light waves. The striking proofs[1] of the admissibility of extending Doppler's principle to moving sources of light, furnished by the astrophysicists in the course of time, gradually silenced their opponents, foremost among whom were Van der Willigen and Spée. We must not omit to mention, however, that an exhaustive theoretical treatment and explanation of the problem has not been given even up to the present time.

The protracted Greenwich observations of the stellar motions, which included forty-eight of the brightest stars, have demonstrated that direct observations with medium sized instruments

[1] I would refer to the introduction to Part I of Vol. VII of the Publications of the Potsdam Observatory, as well as to Dunér's *Recherches sur la Rotation du Soleil.*

cannot furnish results whose uncertainty is of a less order than the average motion of the stars themselves. When the dispersion is just sufficient to permit the definite recognition of the displacement, which at most is slight, the intensity of the spectrum of even the brightest stars in a medium sized instrument is too low to permit even a tolerably accurate measurement. A further reason for the small success doubtless lay in the unsuitability of the apparatus, which was especially lacking in stability.

When I made the first attempt in 1887, with the assistance of Professor Scheiner, to record photographically the displacements of the lines in stellar spectra, and then to measure them as accurately as possible on the spectrograms, it very soon appeared that this constituted a very marked advance in the determination of these motions, which are so significant in stellar astronomy. The accuracy of the observations was increased more than eightfold with the apparatus constructed in 1888; the probable error, which in the Greenwich observations averaged ± 22 km for an evening, being brought down in the Potsdam observations to an average of ± 2.6 km. We may therefore fairly say that the determination of motions in the line of sight thus first received a substantial basis in the spectrographic method, and thereby the widest prospects were opened for a period of new investigations and discoveries.

This success is doubtless due in the first instance to the application of photography; but we must not leave out of consideration that it is also in part due to the fact that a complete departure from previous principles was made in the construction of the apparatus, and an instrument was completed exclusively for this definite purpose, possessing the greatest possible stability. While spectroscopes are even yet constructed so that they can serve for many purposes, — permitting variations in the dispersion, and allowing measurements to be made in the most widely separated parts of the spectrum, — the Potsdam instrument photographs only a small portion of the spectrum in the neighborhood of the hydrogen line $H\gamma$. Its dispersion was so chosen that with sufficient sharpness of the spectrum a difference

between the setting on a line of the star spectrum and one of the comparison spectrum could be determined under the measuring machine with an accuracy corresponding to a motion of but a few kilometers. Where the spectrum of the star permitted, it was further arranged so that not only the position of a single line in the stellar spectrum was referred to that of a corresponding line in the comparison spectrum, but several lines were employed for the determination of the displacement. The accurate identification of the lines in the star spectrum was also attempted during the measurement by direct comparison with a plate of the solar spectrum.

I can restrict myself to these general statements, since the apparatus, as well as the methods of measuring the plates and reducing the observations, are sufficiently well known from the full descriptions given in Part I of Volume VII of the *Publications of the Astrophysical Observatory.* But I desire to add something for the accurate description of the state of this branch of science at the conclusion of our series of observations at the beginning of the decade just past, and I also cannot withhold a critique of our observations from the standpoint gained by wider experience.

I would first point out that several of the precautions taken in the observations, which might appear as carried too far, were conditioned by an unsuitable telescope. The Potsdam 11-inch refractor is of very light construction, and has a wooden tube which occasions a strong effect of temperature in the alteration of the focal length. Although the wooden tube is conical, it has a very appreciable flexure.

In consequence of the slight stability of the telescope, an arrangement had to be thought out for bringing the star accurately upon the slit and holding it there in the proper position during the exposure of the photographic plate. This was accomplished by the simple means of simultaneously observing with a small telescope the image of the slit, illuminated by a Geissler tube, and the image of the star, reflected from the front face of the first prism. This method of guiding

appears to have been adopted later by all observers who have undertaken determinations of velocities by the spectrographic method.

It is no longer indispensable with some of the more recent instruments, as for instance the great Potsdam refractor, since a guiding telescope of nearly the same focal length is attached to the telescope itself.

Although it was my endeavor to develop something entirely new in constructing the stellar spectrograph in 1888, in respect to the prisms I was under the influence of the times, and chose the compound prisms known as Rutherfurd's, which then were considered the most excellent, and in fact possessed the advantage of giving less deviation for the same dispersion, and hence less curvature of the spectral line, and which caused less loss by reflection on account of the less oblique incidence of the rays. Extensive investigations made here at a later time have shown that simple prisms are preferable, and elsewhere the use of prism systems has continually decreased. In observations with the spectrograph we have had the experience that strains occur in the cemented prisms at temperatures below $-2°$, causing a diffuseness of the spectra. A consequence of the above mentioned large flexure of the wooden tube of the 11-inch refractor may be that in many positions of the telescope the collimator and the prism are not fully utilized; except for a slight loss of light, this would have no injurious effect on the observations under conditions of good adjustment, perfect surfaces of the prisms, and complete homogeneity of the masses of glass used for the prisms.

As above stated, however, tensions in the prisms and consequent departures from homogeneity in the glass have shown themselves distinctly at low temperatures, which have caused not only a diffuseness of the spectra and a consequent diminished accuracy, but also have not been without effect on the displacement of the lines. Although this effect may have been exceedingly slight in most cases, it may neverthless have become appreciable in unusual positions of the telescope, such

as occur for stars near the pole, and possibly the large discrepancies for a few stars between our observations and those of other observers may be thus explained. In our observations we have always used the telescope in that position only (east or west) in which the optical axis of the collimator and telescope were adjusted. It was not possible to determine subsequently the effect a of lack of coincidence of the two axes on the displacement of lines in the spectra; this, moreover, could hardly have been determined at the time of observation, since it depends on the changes of temperature in the prisms, and the tension thereby occasioned, as well as upon the declination and the hour angle. The determination of the true temperature of the prisms is in general beset with difficulties, and the determination of the temperature of the separate parts of the prisms is, therefore, hardly to be thought of. We might infer that the errors arising would balance each other in a great number of observations made under the greatest variety of conditions. It would, however, probably be possible to obtain later data as to the errors dependent upon the position of the spectrograph, from a comparison of the Potsdam observations with the results obtained by other observers and with other instruments.[1] Such errors are hardly to be expected in the newer instruments with simple prisms, and having greater stability of the telescope. We have most carefully studied the effect of the temperature on the focal length of the objective of the refractor and upon the focal

[1] At present I am acquainted with determinations of the motions of only something more than half of the objects observed in Potsdam, made by Bélopolsky, Campbell, Newall, and Lord, for the great part of which I am indebted to letters of Messrs. Bélopolsky and Campbell. The comparison of these with the Potsdam observations leads to the following provisional results. The departure of the Potsdam results from those of the above observers averages ± 2.8 km for 23 objects, and the comparison shows with some certainty that the Potsdam values on the average are 1.2 km too large for stars of negative motion, and 0.7 km too small for those of positive motion. This constant departure is more clearly pronounced if we consider Campbell's observations only. Taking the mean from 19 objects the departure V. S.—C. = ± 2.7 km; the negative velocities were found on the average to be 2.5 km larger, and the positive velocities 1.6 km smaller by Vogel and Scheiner than by Campbell. If we apply these average values to the observations, the mean value of the difference V. S. — C. comes out as ± 2.0 km.

length of the objectives of the collimator and camera, as well as the varying dispersion of the prism with the temperature, and we have taken them into account both in the observations and in the reductions of the measures.

The hydrogen spectrum was in nearly all cases used as a comparison spectrum, though attempts were made to use the magnesium line λ 448 $\mu\mu$, which possesses the advantage of being very sharp in some star spectra while the $H\gamma$ line is broad and diffuse; but without success, since this line is diffuse in the spark spectrum. The iron spectrum may also be used, and I have pointed out the advantages gained in the use of this spectrum, as well as that of other metallic substances, have described the method of observation, and have illustrated it by an example—the spectrum of *Sirius*.[1] It would have been better for this purpose to have used the spectrum of *α Cygni* or of a star of the second class, because the lines in the spectrum of *Sirius* are exceedingly fine. For *Sirius* the probable error of the motion in the line of sight obtained from the difference between a single line in the star spectrum and a single line in the comparison spectrum is $\pm$ 1.34 km, so by the use of nine such pairs the probable error of the measurements of the plate would not be more than $\pm$ 0.45 km.

The observations could be made on only forty-seven of the brighter stars, since the spectrum of a star fainter than 2.3 mag. with an exposure of about an hour did not have sufficient intensity to be measured with accuracy. With a longer exposure the practically unavoidable changes in temperature produced such an influence on the spectrograph that the accuracy of the observation was impaired. As a guard against accidental errors it was intended that each star should be observed on at least two nights. At the time when these observations were begun, the possibility that a star might show changes in its motion in the line of sight, in a short time, had not been thought of; yet out of these forty-seven stars four had already been found to have a periodic variation during the progress of the work.

[1] *Sitz. d. k. Acad. d. W. Berlin* **28**, 533, 1891.

The object of this work on the motion of stars in the line of sight, completed in 1891, was first to demonstrate the usefulness of the spectrographic method for instruments of medium size, and further, by means of a thorough and detailed description of methods, to enable an observer equipped with better instruments to carry on observations of this character; and possibly to give the method still further development. There was no need of our repeating these observations in the next few years with the same instrument, while we hoped that in the near future we should be able to extend them with more powerful optical means. Unfortunately the realization of this hope was deferred from year to year, and not until the present year did it again become possible for us to carry on regular observations at the Potsdam Observatory — this time, however, with instruments of the highest order of excellence.

Now, while the seed which we planted ten years ago has not hitherto received in Potsdam the nourishment which we could have wished, yet I have the pleasure of knowing that it has found nourishment in other places, so that it has thriven greatly, and has already blossomed beyond our highest expectations.

In 1890 and 1891 Keeler made his beautiful observations on the radial velocity of the brighter nebulae, by means of a grating spectroscope attached to the great refractor of the Lick Observatory. Fourteen nebulae were examined for motion, and the determinations of these motions are of remarkable accuracy, considering the great difficulty of the observations. The probable error of the result for each nebula, a result which is the mean of several observations, is on the average only ± 3.2 km. Of the fourteen nebulae, nine have a negative and five a positive motion, referred to the Sun. The average motion is 27 km and therefore, if we are justified in drawing a conclusion from so small a number of observations, is of the same order as that of the brighter stars. The greatest velocity — 65 km per second, is that of the well-known planetary nebula *G.C.* 4373, H IV 37. It exceeds by about 10 km that of *a Tauri*, which has the

greatest velocity of any of the brighter stars of the northern heavens.

While making these observations on nebulae, Keeler determined the radial velocities of *α Bootis*, *α Tauri*, and *α Orionis*. He found for the velocity of *α Bootis*, from nine measures during 1890 and 1891, a value of —6.8 km ± 0.3 km; for *α Tauri*, on three evenings, + 55.2 km, and for *α Orionis*, on two evenings + 14.0 km per second. The probable error of a single evening's observations, deduced from the results for the three stars, averages ± 1.8 km.

The Potsdam observations made from 1888 to 1890 give for these three stars the values: —7.6 ± 0.6 km, + 48.5 km, and + 17.2 km. A better agreement, considering the complete independence and entirely different character of the methods used (Keeler made his measures on the D lines), could hardly be expected. With the Lick refractor, which exceeds the Potsdam 11-inch refractor some eight times in light-gathering power, it has therefore been possible to determine the motions of the brighter stars, by direct observation, with about the same accuracy as with the Potsdam refractor by the spectrographic method.

About the end of the year 1891, the great Pulkova refractor of 76 cm aperture was provided with a spectrograph, built exactly according to the model of the Potsdam instrument, and like the latter provided with two Rutherfurd prisms. The instrument was so arranged that the prism box could be removed, and another, containing only a single prism, substituted for it when faint objects were to be observed with small dispersion. Since the refractor had been intended primarily for micrometric work, and only secondarily for spectroscopic, the telescope and observing chair had been designed mainly with reference to the convenience of the observer at the micrometer. For this reason Bélopolsky had to contend with many difficulties before he finally succeeded, after making a number of considerable changes in the telescope and the observing chair, in making spectrographic observations with accuracy. The very unsuitable climate made it necessary for him to abandon his original plan of observing

the motions of the fainter stars, a plan which may be regarded as an extension of the Potsdam observations. He therefore selected special objects from among the variable and double stars for his observing list, and made observations of their motions.

Among his valuable researches should be mentioned his investigations of *δ Cephei*[1] in which he demonstrates the existence of a periodical variation in the velocity of this star in the line of sight, which could be brought into good agreement with the period of its light curve, $5^d\ 9^h$. In the case of *η Aquilae*[2] he discovered a variation in the velocity which found an explanation in the period of light-variability of $7^d\ 4^h$, and finally he found a change in the velocity of *α*[1] *Geminorum*[3] with a period of $2^d\ 23^h.5$. It soon appeared that the line of apsides of this double star system was in rapid revolution, and from careful investigation the period of this motion was found to be 4 years 40 days.[4]

As the interesting double spectrum of the well-known variable *β Lyrae* was known, from spectrographic observations, to contain pairs of bright and dark lines, and since changes in the relative positions of these lines had been discovered which were connected in some way with the period of variability of $12^d.9$, Pickering attempted to calculate the orbit of this hypothetical double star from measures of the distances between the double lines at different phases of the light period. He found a relative velocity of the components of 482 km per second, and a diameter of the orbit, assuming it to be circular, of 85.3 million km. I have more recently pointed out that the distance between the lines, and therefore the relative velocity of the components, has been taken to be too large, by not taking into account a probable overlapping of the lines; since the above value leads to an enormous mass (150⊙) of the system. Bélopolsky has published

[1] This JOURNAL, 1, 160. *A. N.*, 140, 17.

[2] *Mem. Spettr. Ital.*, 26, 101.

[3] *Bull. Acad. St. Petersbourg*, 4, 341. This JOURNAL, 5, 1.

[4] *Mem. Spettr. Ital.*, 26, 101, and 28, 103.

an extensive investigation[1] of β *Lyrae*, based on observations made with the Pulkova refactor, in which are contained many interesting details concerning the changes which take place in the pairs of bright and dark lines during the period of the star's variability. From his measures of the hydrogen line $H\beta$ he found an orbital velocity of 89km per second, a semiaxis of 15 million km, and hence a mass of the system of the same order as that of the Sun.

In 1894, while examining a large mass of observations of β *Lyrae* made with a small single prism spectrograph attached to the photographic refractor of the Potsdam Observatory,[2] I found clearly a certain agreement between the relative displacement of the lines and the period of light variability, but not of the simple character assumed by Béloposky. The variation in the distance between the bright and dark line $H\zeta$, especially, seemed to correspond to a period much longer than the period of the light variation, so that a system of two bodies was no longer adequate to explain the phenomena observed in the spectrum. Now since it has been shown by Myers that the light curve is very satisfactorily represented under the assumption of a double star system, we are compelled to introduce explanations which depend upon phenomena of a physical nature.

Bélopolsky[3] thereupon, in 1897, resumed his investigations and from them there is no doubt that we have come nearer to a decision as to the nature of β *Lyrae*. In his measures he disregarded the bright lines altogether, and restricted himself to the dark magnesium line λ 448 $\mu\mu$, which has no companion emission line, and thus freed them from the influence of a partial overlapping. By so doing he was able to obtain results which were referable to the case of a simple double star without further complication. He found for the orbital velocity 178km per second, for the semiaxis of the orbit 31.9 million km, for the

[1] *Bull. Acad. St. Petersbourg*, N. S., **4**, 341. Mélanges mathem. et astr., Vol. VII, Livr. 3, 1893.

[2] *Sitz. d. K. Akad. d. W. Berlin*, 1894, 115.

[3] *Mem. Spettr. Ital.*, **26**, 135; also Tikhoff, *ibid.*, **26**, 107.

distance between the two components 47.5 km, and for their masses respectively 9 and 18 times the Sun's mass.

Bélopolsky has also found variable motion in λ *Tauri*, ζ *Geminorum*, and θ *Ursae Majoris*,[1] so that his contribution to the discovery of spectroscopic binaries consists of seven objects.

It is of interest to note here the great velocity of —70 km per second, referred to the Sun, which Bélopolsky has found for ζ *Herculis*.[2] Campbell,[3] at the Lick Observatory, has confirmed this observation and obtained a value of 70.3 km, while Deslandres,[4] at Paris, obtained a value about 10 km smaller. In this connection I may remark that Campbell has found in η *Cephei* the greatest known velocity of a star in the line of sight (—87 km per second). This velocity would probably be reduced somewhat if we were to take into account the motion of the solar system. Assuming the coördinates of the apex of the Sun's way to be $a = 267^\circ$ and $\delta = +31^\circ$, and the velocity of the Sun to be 17 km per second, we get for the absolute component of the velocity in the direction of the Sun, —74 km per second for η *Cephei*, —54 km for ζ *Herculis*, and —51 km for the nebula *G.C.* 4373.

In publishing his observations of ζ *Herculis*, Deslandres made a remark concerning the Paris telescope of 1.2 meters aperture, to which his spectroscope was attached, from which it appears that the stability of the telescope is hardly what it should be for such delicate investigations; and, possibly because the mirror did not unite perfectly all the rays on the slit, the exposure had to be made 50 per cent. longer than with the Pulkova refractor, under presumably less favorable atmospheric conditions. In these unsuitable conditions may also be found the reason why so few observations of stellar motion by Deslandres have been made known. He has published beautiful threefold enlargements of the original spectrograms[5] of the four stars α *Aurigae*, β *Aurigae*, α *Canis Majoris*, and γ *Pegasi*, which are specially

[1] *A. N.*, **145**, 281; also **149**, 239, and **151**, 39.

[2] *A. and A.*, Feb. 1894. *A. N.*, **133**, 257.

[3] This JOURNAL, **8**, 157. [4] *C. R.*, **119**, 1252.

[5] *Spécimens de Photogr. astronomiques. Obs. de Paris*, 1897.

remarkable for the great length of spectrum which his apparatus is capable of defining sharply. With the exception of a series of observations of *α Aquilae*, nothing further is known to me of work on the motions of stars at the Paris Observatory.

At the suggestion of Poincaré of Paris, Deslandres has, on the other hand, made spectrographic researches on the motion of the planets, and on the rotation of *Jupiter*,[1] which are of importance. The investigations showed, in accordance with Poincaré's assumptions, that in the case of a body which shines by diffuse reflected light, the displacement of lines in its spectrum depends not only on the motion of the body with reference to the observer, but also on the motion of the body with reference to the source of light by which it is illuminated.[2] The observations on the rotation of Jupiter have been repeated and confirmed by Bélopolsky.

The beautiful results which Keeler obtained by the spectrographic method at the Allegheny Observatory on the rotation of the ring system of Saturn, should be mentioned in this connection. By these observations it was shown that Saturn's rings consist of separate small bodies which revolve about Saturn in obedience to Kepler's laws, and cannot be regarded as a rigid body, thus furnishing a practical confirmation of the conditions demanded by theory.[3] These interesting observations have been repeated by Campbell, Bélopolsky, and Deslandres.

The first attempts to prove the truth of Doppler's principle, by showing that there is a displacement of the lines in the spectrum of the edge of the Sun near the equator which corresponds to the known equatorial velocity obtained from observations of Sun-spots, were made by me twenty-nine years ago. In the

[1] *C. R.*, 120, 417. [2] *A. N.*, 139, 241.

[3] Keeler used in these observations a spectrograph composed of three simple prisms, which produced a total deviation of 180°. From a few original negatives of the solar spectrum and several excellent negatives of planetary spectra, which he kindly sent me, I was able to see that the apparatus is very similar to the Potsdam spectrograph of 1888, both as regards dispersion and resolving power. The dispersion of Keeler's apparatus is about one twentieth greater than that of the Potsdam spectrograph, while the resolution of close lines is almost exactly the same in both instruments. Perhaps Keeler's apparatus is slightly superior to the Potsdam apparatus in this respect.

course of time these observations have been frequently repeated with improved instruments. Dunér, in Lund, has undoubtedly carried out the most thorough investigation of the rotation of the Sun in different zones, by means of spectroscopic methods.[1] He was induced to make these observations because, in the beginning of the eighties, the trustees of the Lars Hjertas Minne endowment expressed a desire to have the spectroscope used for a a careful investigation of the question whether the wave-length of light really varies directly as the velocity of the source of of light, as required by the Doppler-Fizeau principle, and had expressed their willingness to grant sufficient money for a suitable spectroscope.

To give some idea of the results of his observations, which were made during the summer months of the years 1887, 1888, and 1889, I have inserted the following table of the means for various heliocentric latitudes, with their probable errors:

Heliocentric latitude	Velocity in km.	Number of observations	$\xi \cos \phi$	ξ
0°.4	1.98 ± 0.013	107	14°.14	14°.14
15.0	1.85 0.013	104	13.19	13.66
30.0	1.58 0.014	104	11.31	13.06
45.0	1.19 0.014	106	8.48	11.99
60.0	0.74 0.012	107	5.31	10.62
74.8	0.34 0.013	107	2.45	9.34

Shortly before the completion of Dunér's researches there appeared two extended investigations by Crew[2] on the same subject, which led to the result that the absorbing layer of the Sun rotates with a uniform angular velocity, while Dunér's observations cannot be harmonized with the assumption of a constant angular velocity, as is shown by the table, in which the angular velocity (ξ and $\xi \cos \phi$) is given in the two last columns. This is not the place to discuss the difference between the results obtained by the two observers; for it was merely my purpose to mention the beautiful observations which have been

[1] N. C. DUNÉR, "Recherches sur la Rotation du Soleil," *Nova Acta Reg. Soc. Sc. Ups.* Series III.

[2] HENRY CREW, "On the Period of the Rotation of the Sun as Determined by the Spectroscope," *Am. Jour.*, **35**, 151, and **38**, 204.

made in this field, and to show what great accuracy can be given to spectroscopic observations of motion when there is sufficient light. The observations of Dunér agree very well with the law of the rotation of the Sun derived from the observations of Sun-spots.

With regard to the above-mentioned desire of the trustees to ascertain whether the change in the wave-length of light is proportional to the velocity of the luminous source, Dunér's observations have shown that, within the errors of observation, the simple form of Doppler's principle is valid, and the influence of possible higher terms is not recognizable.

Through the generosity of Mr. D. O. Mills, Professor Holden, at that time Director of the Lick Observatory, was able to have a spectrograph constructed for the Observatory about the middle of the nineties, to be used exclusively for determining stellar motions in the line of sight. In the October 1898 number of the ASTROPHYSICAL JOURNAL Professor Campbell has given an extended description of this instrument, which, mainly through his endeavors, has become the most noted instrument of its time.

Attached to the great 36-inch refractor of the Lick Observatory, used under the most excellent atmospheric conditions, and in the hands of a circumspect and careful observer, the "Mills spectrograph" has in the last few years achieved surprising results. The spectrograms taken with it possess, on the average, a sharpness which excels that of even the best spectrograms taken with the Potsdam apparatus of 1888, and the resolving power of the instrument is much greater. Through the kindness of Messrs. Keeler and Campbell I received last year two original negatives, one of γ *Andromedae*, and the other of η *Pegasi;* so that I was able to thoroughly assure myself of their excellence. They are marked as being above the average exellence, and their measurement is a real pleasure; yet even with such plates great care and long experience are required to obtain uniformly consistent results.

With spectra which contain many lines, and which differ considerably from the solar spectrum, the use of a solar spectrum in

measuring the plates cannot well be avoided. In many spectra, on the other hand, especially those with strong iron lines, the stellar and comparison spectra can be compared directly by means of Hartmann's interpolation formula.[1] The amount of the shifting of a known line in the star spectrum can be exactly determined even if there is no identical line in the comparison spectrum; for instance, the lines of hydrogen and clèveite gas can be compared with iron lines in the comparison spectrum.

On Campbell's plates the probable error of the determination of the distance between a stellar line and the corresponding comparison line may be taken as ± 1.2 km, and since from ten to twenty measurable lines may easily be found in the spectrum of a star of class II, the probable error of the mean of the measures on a single plate may be reduced to about $\pm \frac{1}{3}$ km. Slight changes in the instrument during the exposure, slight differences in the observer's habit of measurement, minute distortions of the film, and other unavoidable sources of error have here to be taken into account. They are recognizable in the fact that the probable error of the final result, deduced from the mean of several plates of the same object, is larger than that which the probable error of a single plate, determined from the agreement of the different lines, would lead one to expect. According to Campbell's observations of the brighter stars, the probable error of a single plate is however somewhat less than ± 1 km. Campbell's detailed description of the Mill's spectrograph, and the opportunity I have had of examining some of the spectrograms made with it, have been of great service to me, since the new spectrographs for the large Potsdam refractor, which have been made from my designs by Toepfer, of Potsdam, were just at that time so nearly completed that they could be subjected to a preliminary test. The construction of these two spectrographs was begun in 1897. One of them has three simple prisms and a total deviation of 180°, while the other has only one simple prism. Dr. Hartmann, whom I had commissioned to make the adjustments and tests of the optical parts of

[1] *Publ. des Astrophys. Obs. zu Potsdam*, **12**, No. 42.

these spectrographs, has devoted all his energies to obtaining the highest degree of excellence,[1] and it is a pleasure to be able to say that the new spectrograph with three prisms for the Potsdam 80 cm refractor, will not be inferior to the Mills spectrograph.

At the present time a series of observations of the radial velocities of stars down to the fifth magnitude is being made at the Lick Observatory. So far Professor Campbell has made two or more observations on about 300 stars, and has found 16 of them to have a variable velocity, thus bringing the number of binaries discovered by spectrographic methods up to 28. I shall merely give here a list of the spectroscopic binaries discovered by Campbell.[2]

Star	Period
η *Pegasi*	2¼ years
χ *Draconis*	9⅓ months
o *Leonis*	14½ days
ζ *Geminorum*	Unknown
ι *Pegasi*	Over 10 days
θ *Draconis*	Over 9 days
ϵ *Librae*	Unknown, several months
β *Capricorni*	Unknown, long
h *Draconis*	Undetermined
λ *Andromedae*	About 20 days
ϵ *Ursa Minoris*	A few weeks
ω *Draconis*	Unknown
α *Ursa Minoris*	3.9 days and a second longer period.
α *Aurigae*	3½ months
ϵ *Sagittarii*	A few weeks
β *Herculis*[3]	Unknown, one year?

It is worthy of remark that several stars have a period of many months, and that η *Pegasi* has a period of 2¼ years. Thus the gap which formerly existed between the spectroscopically discovered and the visual double stars, with respect to the lengths of their periods of revolution, has been filled up.

When we consider the large number of variable stars of the *Algol* type which have been discovered photometrically during

[1] For particulars on this investigation, and the methods used in testing, see Dr. Hartmann's paper in this and the next numbers of this JOURNAL.

[2] This JOURNAL, 8, 9, 10. [3] *A. S. P.*, 12, 39, 1900.

the last decade, and for which the assumption appears to be justified that their variation is a consequence of duplicity ; when we further consider that these stars can be recognized as variable only when the line of sight makes an extremely small angle with the plane of the orbit, and that for the spectroscopically discovered stars, also, this angle cannot be supposed to be very great, we cannot repress our astonishment at the rapidity with which the number of stars thus discovered has grown.

Among the double stars discovered at the Lick Observatory, *Polaris*, which has a double period of motion in the line of sight, is of particular interest, since we are here led to infer the existence of three bodies. The discovery of the short period is an admirable proof of the excellence of the observations, since the variation from the mean velocity is only ± 3 km.

Of particular interest, also, is the discovery of the periodic doubling of the lines in the spectrum of *α Aurigae*, which consists of two superposed spectra uniting at times to form a spectrum strongly resembling that of the Sun.[1] Although *α Aurigae* was frequently observed at Potsdam during the years 1888–1891, this peculiarity of its spectrum escaped us, and the explanation of the fact that most of our spectrograms of *α Aurigae* are ill-defined, and the lines quite extraordinarily broadened, was only given later, by the investigations of Campbell. The days on which the spectrograms obtained were good and sharp, are those on which the two spectra were superposed, and our early observations have now at least given us the means of arriving at a more accurate value of the period ($104^d.1 \pm 0^d.2$). By means of observations with the new Potsdam refractor and

[1] From Campbell's observations the deduction can also be made that both stars, one having a spectrum similar to the solar spectrum, and the other a spectrum containing the hydrogen and the stronger iron lines, are of nearly the same mass, since their displacements from the mean position are about equal. Assuming the period to be $104^d.1$ and the maximum relative velocity to be ± 30 km, and assuming provisionally that the orbit is nearly circular, with its plane in the line of sight, I have found, for the combined mass of the stars, $m + m_1$, $= 2.3 \odot$ and for their distance, 85.3 million km.

spectrograph, we have been able to confirm Campbell's observations, both of *α Aurigae* and *α Ursae Minoris*.[1]

When we consider that researches on the motions of stars in the line of sight have been begun and are being carried on with good results by Newall, in Cambridge, and by Lord, at the McMillin Observatory in Ohio; that in Meudon a double refractor of the same size as the Potsdam instrument has been set up, and provided with a spectrograph, with which Deslandres[2] has already succeeded in showing that *δ Orionis* is a star with variable velocity; that further, the largest instrument in the world, Yerkes refractor at Williams Bay, will also be used for this purpose, and that Gill, with the double refractor of the Observatory at the Cape of Good Hope, will extend the same researches to the southern heavens, we may confidently expect that our knowledge of the stellar system to which we belong will be increased as much in the new century upon which we are entering, as our knowledge of the solar system has been increased in the century just past. The energy with which some of the largest observatories in the world are participating in the work is most encouraging, for the amount of work to be done has grown most unexpectedly during the last decade, through the discovery of so many stars having a variable velocity in the line of sight.

In closing this review, I should not wish to leave unmentioned the fact that in one case the application of the Doppler-Fizeau principle can no longer be regarded as valid; I refer to the interpretation of the pairs of bright and dark lines which are found in the spectra of the new stars. When this peculiarity of the spectra of new stars was discovered, it was very natural that the relatively strongly displaced emission and absorption lines should be ascribed to the spectra of two bodies, whose motions in the line of sight were oppositely directed. Corresponding to the great displacement of the lines, velocities were

[1] It should be added here that the discovery of the duplicity of the lines of *α Aurigae* was made independently, and almost simultaneously, by Newall, of Cambridge, England.

[2] *C. R.* 130, No. 7.

arrived at which, in comparison with the mean velocity of other heavenly bodies whose motions have been spectroscopically determined, must be characterized as perfectly enormous — more particularly since it was improbable that the whole motion of the bodies should be in the line of sight.

But since in the stars which have appeared more recently, *Nova Normae* and *Nova Carinae*, the same pairs of bright and dark lines were found, in which, as in the spectrum of *Nova Aurigae*, the emission line lay on the less refrangible side; and since similar lines can be observed in the spectra of *β Lyrae* and *P Cygni* (the Nova of 1600); and particularly since no change in the distance between the bright and dark lines could be detected during the whole of the first apparition of *Nova Aurigae*, and the lines remain unchanged in position in the spectrum of *P Cygni* — while in *β Lyrae*, though they vary during the star's period, the changes are never so great as to reverse the positions of the dark and bright lines — doubts as to the applicability of Doppler's principle to such cases appeared to be more and more fully justified.

The assumption that we are here concerned with phenomena of a purely physical nature gained a firmer basis through the researches of Humphreys and Mohler, Eder, and Wilsing,[1] on the changes produced in spectral lines by high pressure. It was found that under high pressure pairs of bright and dark lines could be produced in metallic spectra, in which the emission line always lay on the less refrangible side.

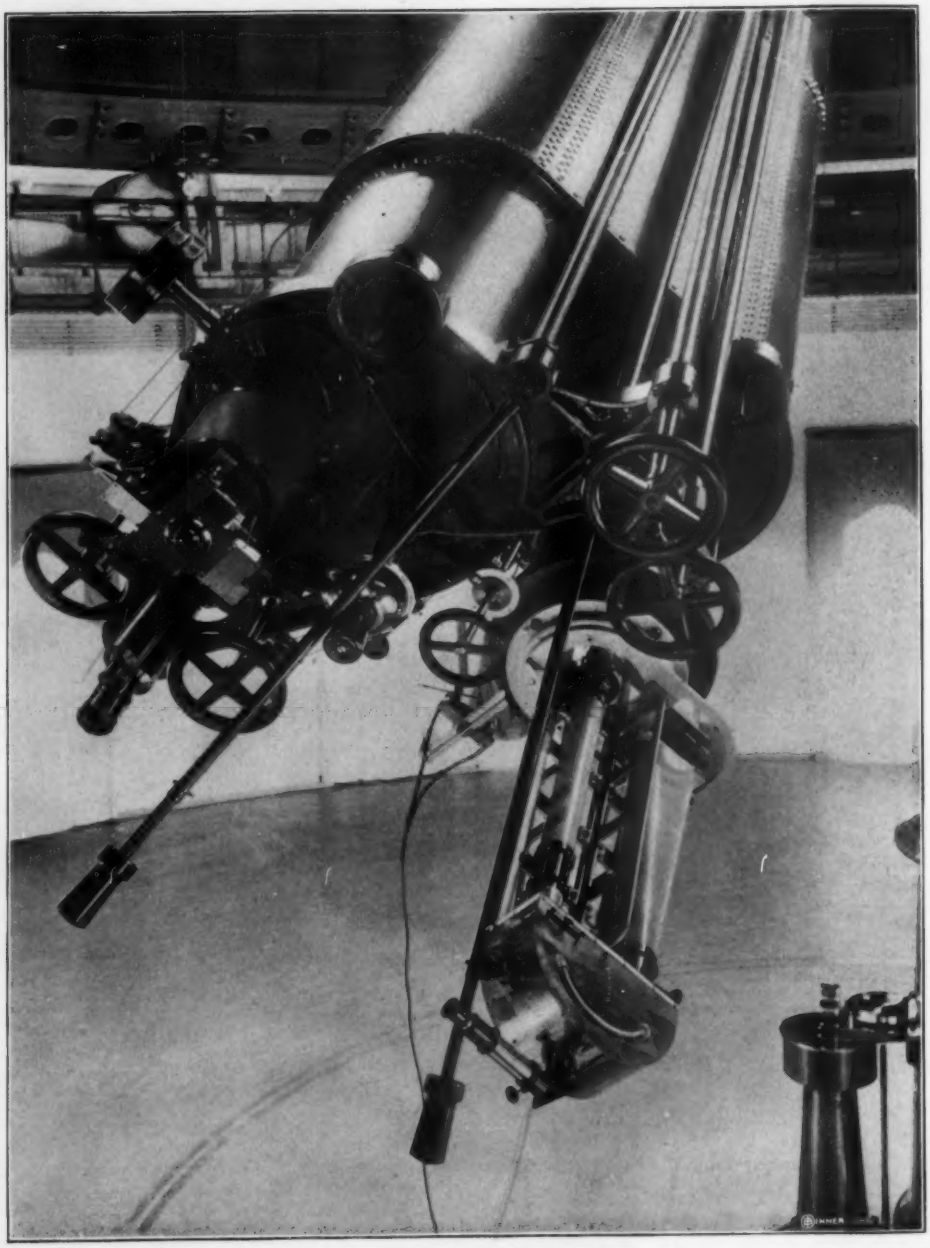
These observations are to be regarded as only first beginnings; but doubtless through them a wide field of highly interesting research is opened, in which the astronomer may hope for the zealous support of the physicist. The veil which has enveloped anew our knowledge of the nature of the temporary stars will certainly be lifted, when experiment shall yield results which harmonize with the phenomena observed in them; and not until then will the time come to frame hypotheses respecting

[1] "Ueber die Deutung des typischen Spectrums der neuen Sterne." *Sitz. d. K. Akad. d. W. Berlin*, **24**, 425. 1899.

the origin of the abnormal conditions of pressure in their atmospheres.

It is quite possible that, as observations of motions in the line of sight become more and more refined, the conditions of pressure in the atmospheres of the stars can no longer be neglected, although it may be assumed that the pressure in the atmospheric layers from which the light comes to us is neither subject to great variations for any individual star, nor varies widely for different individuals. Even if this should not be true, the means of arriving at accurate results for stellar motion, and at the same time of gaining information concerning the conditions of pressure in the stellar atmospheres, is to be found in the use of different metals for comparison spectra.

PLATE IX



NEW POTSDAM SPECTROGRAPH (III)

DESCRIPTION OF THE SPECTROGRAPHS FOR THE GREAT REFRACTOR AT POTSDAM.

By H. C. VOGEL.

As I HAVE already mentioned in the preceding article, two new spectrographs have been constructed for the Potsdam 80 cm refractor, and I shall give here a preliminary description of some further details of these instruments, which will serve to make more intelligible the illustrations in Plates IX, X, and XI.

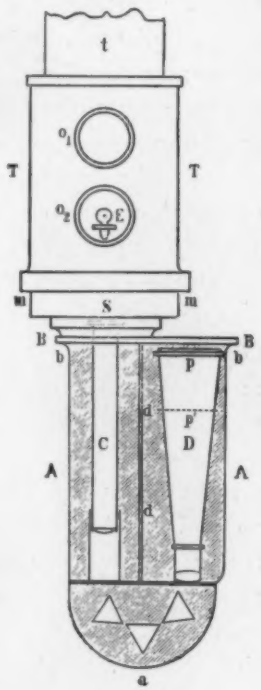


FIG. 1.

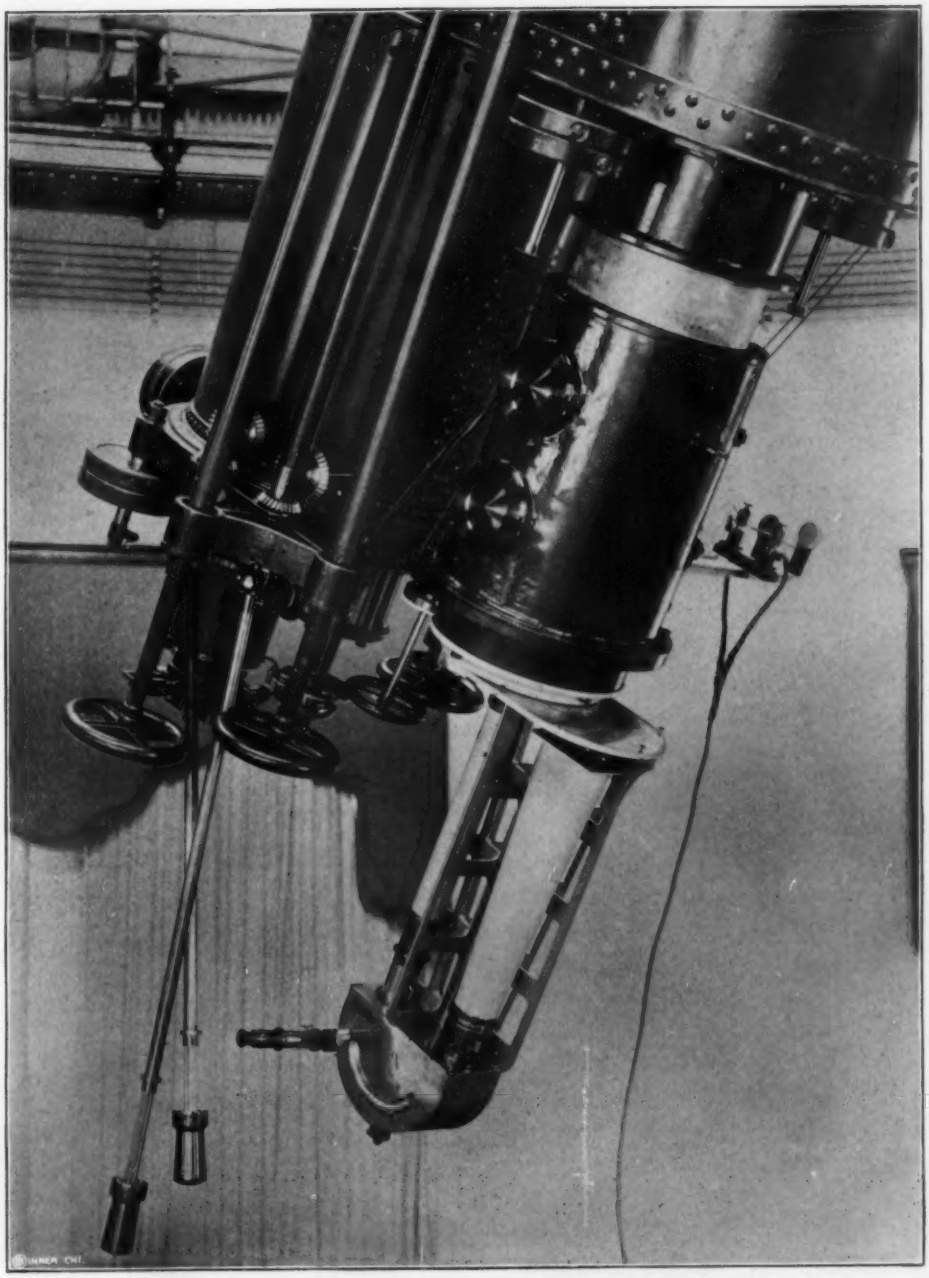
In the construction of the larger spectrograph with three prisms, I have departed somewhat from the previously chosen form, in order to obtain greater stability by firmly uniting collimator, prisms, and camera by means of a single plate. The steel plate *A* (see the schematic diagram in Fig. 1) has a thickness of 7 mm and a length of 78 cm. The width at *bb* is 41 cm, while at *a* the plate is rounded, to correspond with the arc formed by the prisms. This plate is rigidly connected with the elliptical base-plate *BB*, which is 10 mm thick. To guard against lateral flexure, the plate *A* is provided with three ribs, one, *dd*, in front, 12 cm deep and 5 mm thick, and two, on the back, 8 cm deep and 5 mm thick. The steel plate *BB* is secured by strong screws to the cylindrical casting *T*, which is rigidly connected with the draw-tube *t*, at the eye-end of the photographic telescope. *C* is the collimator, *D* the camera, and *p* the plate-holder. The camera *D* can be exchanged for another, which has an objective of shorter focal length. When this second camera is

used, its objective is of course as close as possible to the last prism, and the plate-holder has the position p'. The slit S projects somewhat into the cylindrical piece T. It has the same construction as the slits in all our spectrographs; namely, one movable jaw, which is pressed against the fixed jaw by a spring, and is separated from the latter by turning a micrometer screw. The jaws are made of platinum-iridium. The arrangement for photographing comparison spectra in juxtaposition to the star spectrum, and for limiting the length of the slit, has been described by Dr. Hartmann.[1] I should add that the small plates or screens have been placed at a small distance in front of the slit, in order that their boundaries may be sharply defined on the photograph, which is not the case when they are placed in contact with the slit-plate. At *mm* is an arrangement allowing the apparatus attached to T to be rotated around the optical axis of the refractor. The angle of rotation can be read to $1'$, and in this way the slit can be accurately placed in any desired position angle.

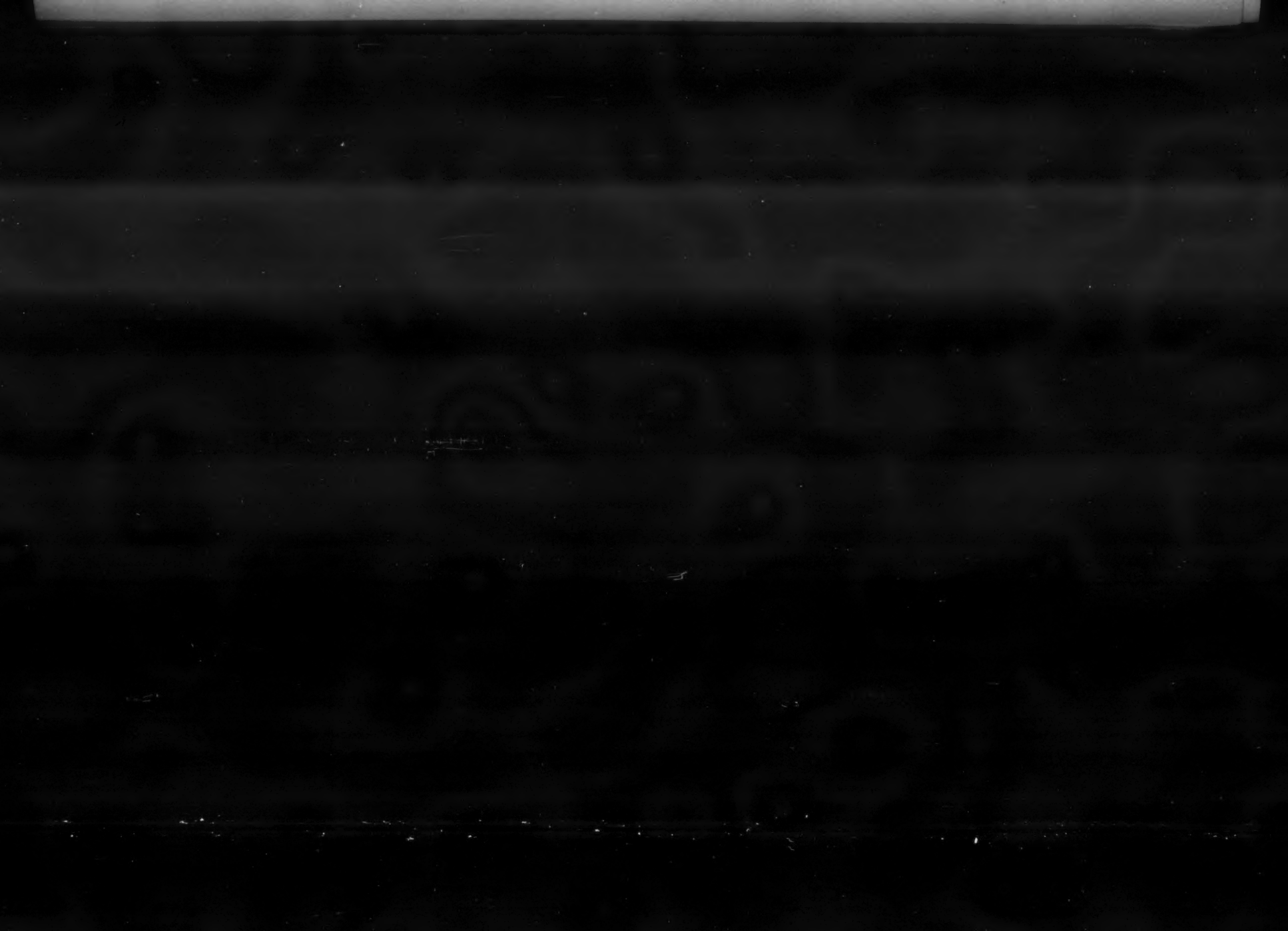
In the cylinder T, which has a length of about 57 cm and a diameter of 36 cm, there are four apertures, closed by means of covers (in the diagram, only o_1 and o_2 can be seen). These apertures serve to introduce Geissler tubes, or other apparatus which it may be necessary to place in front of the slit. At E is the (electric) source of light, the rays from which are thrown on the slit by a mirror placed, inside the cylinder T, in the optical axis of the refractor and the collimator, when the comparison spectrum is to be photographed. Dr. Hartmann has found that the arc light is advantageous for producing the comparison spectrum. The electric plant for turning the dome, opening the shutter, and moving the observing chair furnishes a current which suffices for the arc light, and which is easily led from the observing chair to the instrument. An exposure of about one minute produces a sufficiently intense comparison spectrum when (and this is now always done here) the light is rendered diffuse by interposing a translucent screen between the source of

[1] *Zeitschrift für Instrumentenkunde*, 20, 57, February 1900. This JOURNAL, July 1900.

PLATE X



NEW POTSDAM SPECTROGRAPH (III)



light and the slit. The effect of this screen is to prevent entirely any accidental displacement of the spectral lines, in the case of a possible imperfect adjustment of the cone of rays from the source of light with respect to the optical axis of the collimator.

The collimator objective, by Steinheil, has a focal length of 48 cm; $\frac{A}{F}=\frac{1}{15}$. The equivalent focal length of the camera objective (an anastigmat by Zeiss, of Jena) is 56 cm; $\frac{A}{F}=\frac{1}{14}$. The second camera objective (a cemented triple lens by Steinheil) has a focal length of 41 cm; $\frac{A}{F}=\frac{1}{10}$.

The prisms were made by Steinheil of very white Jena glass *O.* 102, ($n=1.6744$ for $H\gamma$). The refracting angle of each is $63^\circ\ 27'.5$, the length of the refracting edge (height of the prism) is 40 mm. The lengths of the refracting faces of the three prisms are respectively 60 mm, 65 mm, and 70 mm, and the total average length of path, when the prisms are in the position of minimum deviation for the $H\gamma$ rays, is 103 mm. The anastigmat lens of the first camera makes it possible to obtain a uniformly sharp spectrum from *b* to *K*, but the strong curvature of the lenses causes a somewhat greater loss of light with this camera than with the second objective of 41 cm focal length.

For the sake of lightness, numerous openings are left in the steel plate *AA* and its strengthening ribs. The steel parts were finished with a fine matt surface and then nickel-plated. The box which encloses the prisms is made of thin sheet nickel, and is highly polished, in order to protect the prisms as much as possible from radiant heat, and during the observations the apparatus is further protected by enveloping it in cloth. A telescope for observing the image of the star in the slit, formed by light reflected from the first surface of the first prism, is provided, as in the earlier apparatus, and provision is also made for so connecting this telescope with the prism box, that the image

reflected from the first face of the second prism — that is, the star spectrum formed by the first prism — can also be observed. In the prism box is a thermometer, the bulb of which projects into the box, while for further determinations of temperature there is a thermometer at the lower end of the tube of the refractor, and another near the great objective, both within the tube. They can be illuminated by incandescent lamps, and are easily read from the eye-end of the telescope by appropriate systems of lenses.

The weight of the spectrograph (which has been designated by the numeral III), is 31 kg.

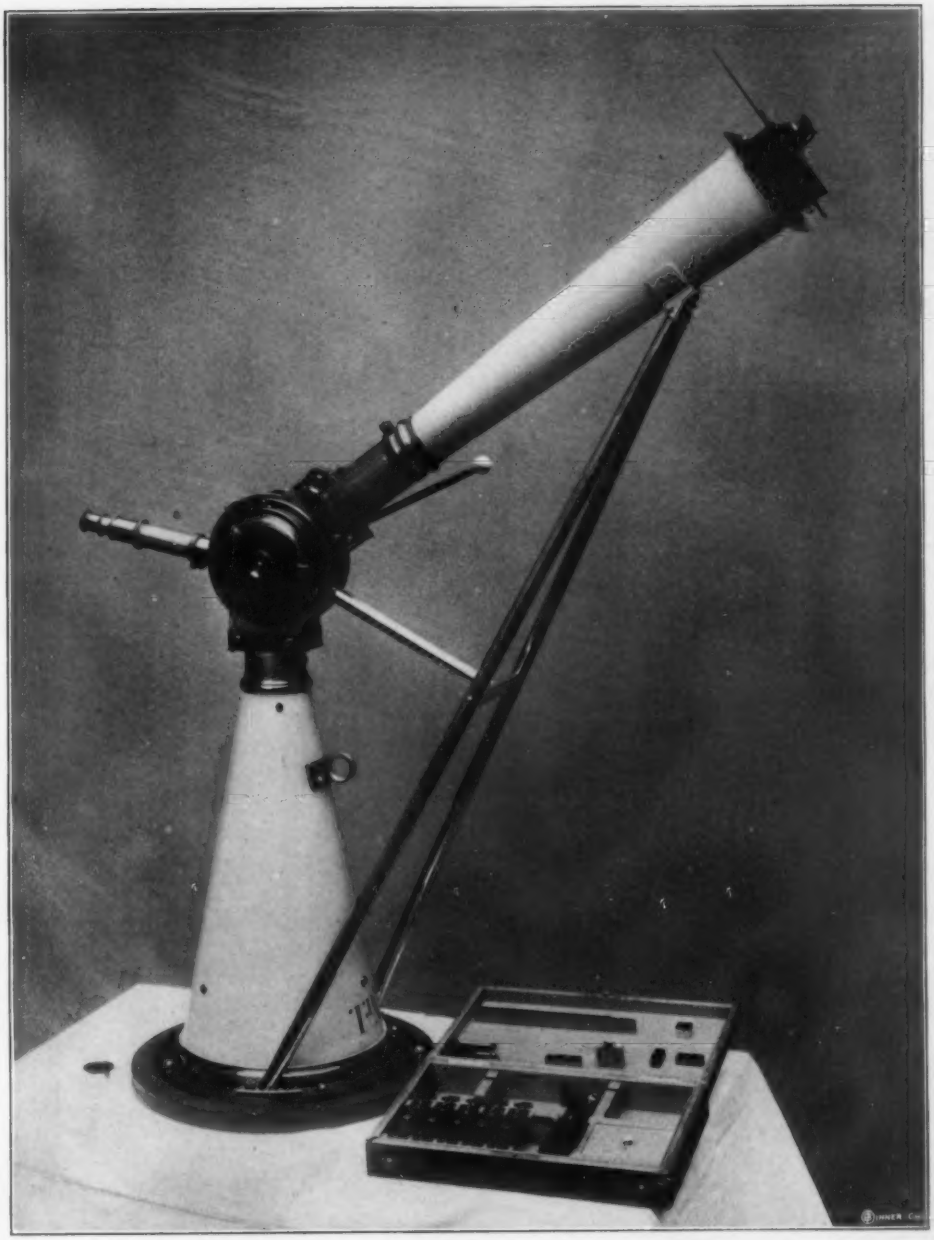
The construction of the apparatus I, which is provided with only a single prism, will be readily understood from the illustration. The focal length of the collimator objective is 53 cm; $\frac{A}{F} = \frac{1}{15}$. The focal length of the cemented triple camera objective is 72 cm; $\frac{A}{F} = \frac{1}{18}$. Both objectives are from the shops of C. A. Steinheil's Sons, Munich, while the prism, which has a refracting angle of 60°, a height of 45 mm, and faces 61 mm long, was made by Zeiss, of Jena. This apparatus gives a sharp spectrum from D to N, corresponding to a difference of wave-length of 230 $\mu\mu$. The weight of the apparatus is 20 kg.

The mountings of both instruments were made in the most satisfactory manner by the mechanician, O. Toepfer, of Potsdam.

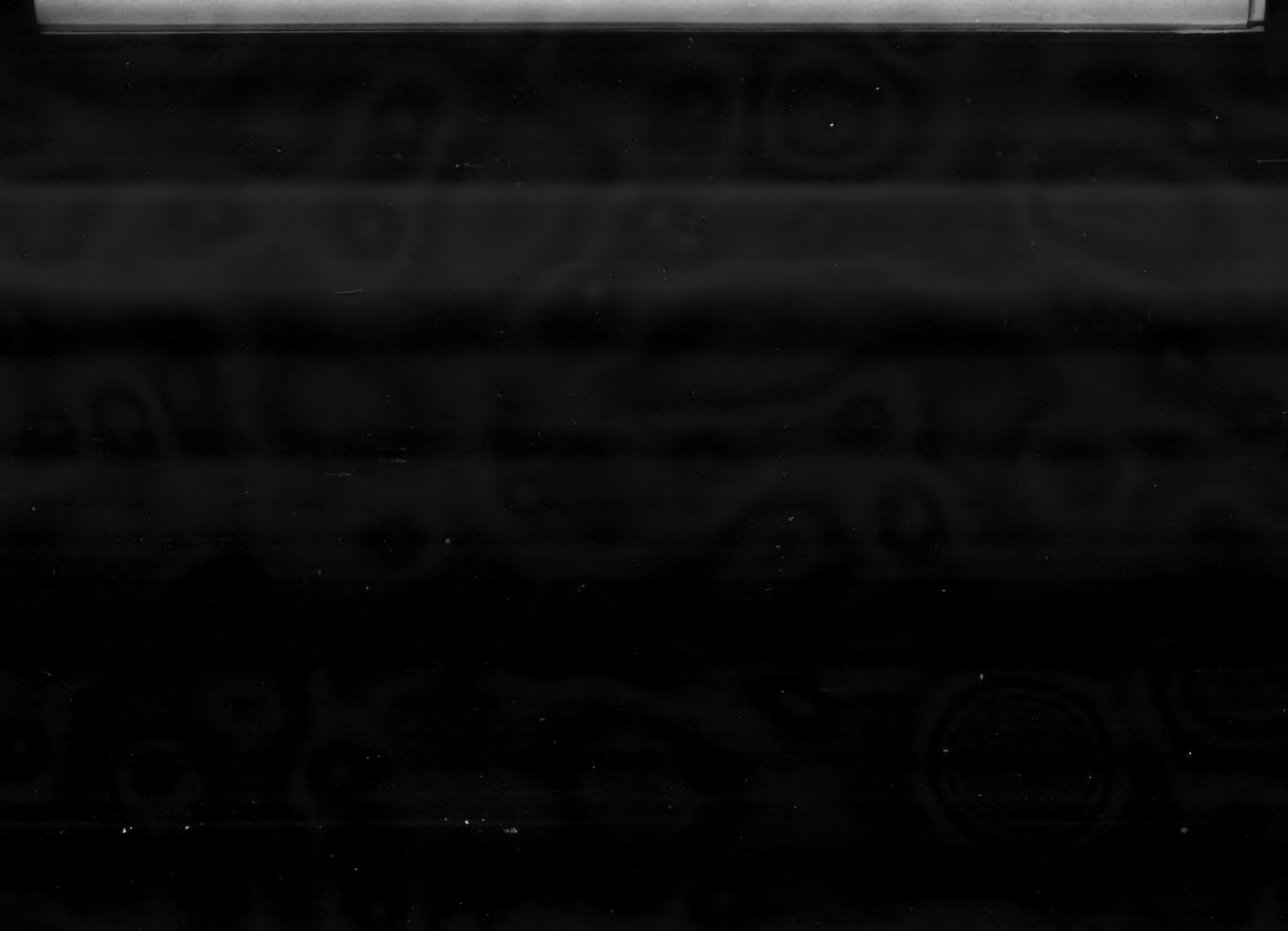
The fine groups of lines in the solar spectrum, which Campbell, in his description of the Mills spectrograph, has cited as a test for such instruments, are perfectly resolved with the same angular slit-width.

In order to permit a more accurate judgment of the capabilities of the apparatus to be made, with respect to the resolution of close lines on a photograph of the solar spectrum, I give the following table of results:

PLATE XI



NEW POTSDAM SPECTROGRAPH (I)



Region above K

λ	Δ	
3921.563	0.132	Very distinctly divided; spectrum not quite sharp.
3921.695	0.160	
3921.855		

Region between H and K

λ	Δ	
3935.965	0.156	Widely divided
3936.121		
3938.439	0.113	Distinctly divided
3938.552		
3948.818	0.107	Well separated
3948.925	0.114	
3949.039		
3952.465	0.084	Very weak lines; divided
3952.549		
3952.754	0.096	Indications of resolution
3952.850		
3956.476	0.127	Well separated
3956.603		

Between H and K all lines in Rowland's atlas of the solar spectrum can be seen.

Below H

λ	Δ	
4056.135	0.086	Only four lines can be recognized; the first two are not resolved, the others distinctly
.221	0.124	
.345	0.150	
.495	0.106	
.601		
4074.835	0.112	Resolved
.947		
4076.644	0.148	Very well divided
.792		
4078.515	0.116	Not completely resolved, though duplicity can be distinctly recognized.
.631		
4092.431	0.116	Resolved though partly blended together
.547	0.118	
.665	0.156	
.821		

Between G and *Hγ*

λ	Δ	
4311.058	0.088	Not divided
.146	0.182	Divided
.328		

Between G and *Hδ*

λ	Δ	
4311.608	0.066	Not divided
.674	0.206	Divided
.880		
4314.248	0.133	Divided
.381	0.098	Not divided
.479		
.964	0.174	Divided
4315.138	0.124	Not divided
.262		
4320.907	0.212	Completely divided
4321.119		
4339.617	0.265	Widely divided; easy pair
.882		
4344.451	0.219	Distinctly divided
.670		
4351.930	0.153	Indications of resolution
4352.083		
4408.364	0.218	Well divided
.582	0.101	Not divided
.683		
4530.910	0.213	Three lines just recognizable
4531.123	0.204	
.327		
4533.133	0.286	Distinctly double
.419		
4534.139	0.201	Indications of the companion
.340		
4535.741	0.138 Not div'd	Appear like double line
.879	0.215 Divided	
4536.094	0.128 Not div'd	
.222		
4540.672	0.208	Divided
.880		
4556.063	0.243	Divided
.306		
4565.688	0.154	Indications of resolution
.842		
4607.510	0.321	Well divided
.831		
4613.386	0.158	Apparently single
5.44		

All the tests which have been made so far with respect to the stability of the apparatus have given very satisfactory results, so that there can be no further doubt that the Potsdam instrument will yield results equal to those obtained with the Mills

spectrograph. On account of the longer collimator of the Mills spectrograph, corresponding to the greater focal length of the Lick refractor, this instrument has an advantage, characterized chiefly by the shorter exposure times required, in that the slit can be opened more widely than that of the Potsdam instrument, for equal resolution of the systems of lines in the spectrum. On the other hand, the anastigmat lens of the longer camera, and the fact that the 80 cm objective of the Potsdam refractor is achromatized for the chemically active rays, permit a much greater length of sharply defined spectrum to be photographed with the Potsdam apparatus, a circumstance which is of great importance for investigations which are not specially restricted to the determination of motions in the line of sight.[1]

Taking absorption into account, the ratio of brightness for the Lick and the Potsdam refractors may be taken to be 11:9, or, when the correcting lens, which does not reduce the aperture, is used with the Lick refractor, 7:6. With respect to brightness the Lick instrument has therefore no material advantage, if we leave out of consideration the favorable atmospheric conditions on Mt. Hamilton. But in changing from a refractor of 5.4 m focal length to one of 12 m, we have already had ample opportunity to appreciate how weighty are these very advantages of transparent and quiet air with respect to the decrease in the requisite times of exposure. The observer with the Mills spectrograph on the Lick telescope will therefore have an advantage, particularly in consequence of superior atmospheric conditions; and will always have it. I am, however, convinced that in choosing an instrument with the relatively short focal length of 12 m, we have obtained that which is best suited to our own atmospheric conditions, and have secured the greatest possible efficiency.

In further comparison of the instruments in question I have compiled the following table, in which are given the values corresponding to a displacement of 0.25 mm (one revolution of the measuring apparatus used at Potsdam and Mt. Hamilton):

[1] The Lick refractor has recently been provided with a correcting lens for uniting the chemically active rays on the slit of the spectrograph. Its use is, however, attended by some further loss of light.—EDITORS.

	μμ	km
Potsdam spectrograph (1888)	0.324	224
Mills spectrograph	0.3143	217
Potsdam spectrograph III (1898), camera 1	0.2546	176
Potsdam spectrograph III, camera 2	0.344	238
Potsdam spectrograph I	0.675	466

For the derivation of the formula for spectrograph III Dr. Hartmann has made the following measures on the solar spectrum:

R	Wave-length (Rowland)	Wave-length (computed)	O.—C.
— 0.224	4654.74	4654.76	— 0.02
+ 17.220	4590.13	4590.13	0.00
34.180	4531.80	4531.83	— 0.03
51.576	4476.22	4476.22	0.00
65.888	4433.39	4433.38	+ 0.01
86.404	4376.11	4376.10	+ 0.01
100.000	4340.63 $H\gamma$	4340.61	+ 0.02
108.706	4318.82	4318.83	— 0.01
136.228	4254.50	4254.50	0.00
156.776	4210.53	4210.53	0.00
178.468	4167.44	4167.45	— 0.01
194.764	4137.16	4137.14	+ 0.02

The formula

$$\lambda = 3279.50 + \frac{[7.762974]}{(R + 595.550)^{\frac{1}{2}}},$$

computed from these data represents, as the last two columns of the table show, all the observations to within the limits of error of measurement. The region of the spectrum here represented includes the entire range of 50 mm, symmetrical with respect to $H\gamma$, which is measurable at once with the screw of the new measuring apparatus.

A formula for spectrograph I has already been published.[1] Expressed in the terms of the units here adopted it is,

$$\lambda = 2232.8 + \frac{[5.819004]}{R - R_0},$$

and represents accurately the part of the spectrum between λ 3709.6 and λ 5183.7.

[1] *Pub. Astroph. Obs. Potsdam*, **12**, No. 42.

REMARKS ON THE CONSTRUCTION AND ADJUSTMENT OF SPECTROGRAPHS. I.

By J. HARTMANN.

AT the suggestion of Professor H. C. Vogel I made a series of special investigations some two years ago, the results of which have been adopted for the construction and adjustment of two spectographs which are to be employed in connection with the large refractor for the photography of stellar spectra. Since the apparatus, which has now been completed for some time, has proved to be extremely efficient and reliable, I desire to communicate here some of the results of those investigations which may also be of service elsewhere in the construction of spectroscopic apparatus.

I. CHOICE OF OBJECTIVES FOR COLLIMATOR AND CAMERA.

Hitherto the objectives for the collimator and camera of a spectroscope have commonly been selected from types of lenses already existent; for the collimator objective a telescope objective is usually chosen, and a similar lens for the camera, or else one of the various systems of photographic objectives now found in commerce.[1] But the properties of all these lenses only partially satisfy the demand which they must fulfill in spectrographs, and in many cases the efficiency of the instrument could be increased by the construction of objectives especially calculated for this purpose. In now giving a few brief remarks on the essential properties of these lenses, I hardly need to remind the reader that only especially clear and transparent kinds of glass can be employed in making the lenses, and their thickness is to be limited to a minimum.

[1] Extended investigations have been published by others as to the best choice of apertures and focal lengths. See Wadsworth, "The Modern Spectroscope." This JOURNAL 3, 321, 1896; Keeler, "Elementary Principles Governing the Efficiency of Spectroscopes for Astronomical Purposes." *Sidereal Messenger*, 10, 431, 1891.

The demands made upon the collimator objective and the camera objective are decidedly different.

The collimator objective must render exactly parallel all the rays of different wave-lengths which proceed from the slit, that is, from points lying very near to the principal optical axis. If an *astronomical* objective is employed, *i.e.*, one computed only for points on the axis and for parallel incident light, it must therefore be as free as possible from spherical aberrations, and must be especially well achromatized if it is desired to photograph a considerable extent of spectrum with the constant length of collimator; on the other hand a large angular aperture is not necessary.

But for the camera objective a large angular aperture is necessary in addition to as complete suppression of spherical aberration as possible, while the achromatism comes in as a secondary consideration. By giving the photographic plate a definite inclination to the axis of the camera it is possible to bring the sensitive film simultaneously into the focus of rays of different wave-length, even when their focal lengths are *not* equal, but only increase uniformly with the deviation of the rays in question. Now a uniform increase of this sort is quite as well possible for a lens which is not achromatic as for one which is achromatized for a point of the spectrum considerably distant from the region which is then to be photographed.

For a photographically corrected lens the favorable case may occur that the blurring of the image is compensated by the residual difference of focus for the rays of different wave-length, which usually produce the secondary spectrum. As is well known the focus of achromatic objectives composed of two kinds of glass reaches a minimum for some spectral ray lying in the middle of the portion achromatized. If the photographic plate is now placed vertically to this ray, all the other rays falling on the plate will have to cover longer paths from the objective to the plate, and since the focus is longer for these, as already mentioned, we may by properly achromatizing the objective accomplish the result of uniting these lateral rays precisely on the plate.

It is a very difficult problem for opticians to construct a lens fulfilling the above conditions—angular aperture of about 20°, absence of spherical aberration with the greatest possible aperture, and, for given dispersion, position of all the foci on a straight line not too greatly inclined to the axis.

Permit me to point out a theorem which results when a simple plano-convex lens from the same glass as the prism is chosen for the camera objective.

If we let r be the finite radius of curvature of a plano-convex lens, n be the index of refraction for any desired ray in the common material from which the lens and prism are made, A be its deviation and b the angle of the prism, and finally F be the focal length of the lens for the rays in question, then for the prism at the minimum deviation we shall have the well-known relation

$$n = \frac{\sin \frac{1}{2}(A+b)}{\sin \frac{1}{2} b} = \sin \frac{A}{2} \cot \frac{b}{2} + \cos \frac{A}{2} .$$

If we introduce this expression in the formula applying to the plano-convex lens

$$F = \frac{r}{n-1} ,$$

we shall obtain the following relation not involving n:

$$F = \frac{r}{\sin \frac{A}{2} \cot \frac{b}{2} - \left(1 - \cos \frac{A}{2}\right)} .$$

This is the polar equation of the focal line of the system in question, the pole being placed at the optical center of the lens. The term $1 - \cos \frac{A}{2}$ vanishes for small values of A, and since

$$F = \frac{r}{\sin \frac{A}{2} \cot \frac{b}{2}}$$

is the equation of a straight line which runs parallel to the incident ray at a distance of $r \tan \frac{b}{2}$, it follows that the foci of the

system will lie very nearly in a straight line for not too large deviations, so that considerable stretches of spectrum can be rendered simultaneously sharp by inclining the plate-holder to the axis of the camera.

I show the relations for a prism of 60° in Fig. 1. *O* is the optical center of the camera lens, whose radius of curvature is

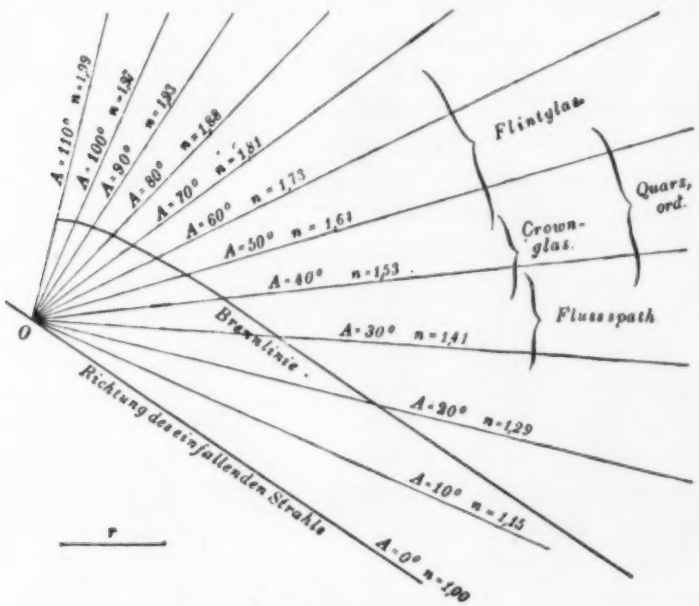


Fig. 1.

represented by the distance *r*. It will be seen that the focal line approximates a straight line quite closely for all deviations which will practically occur. The relation here derived will be especially useful in research with a quartz spectrograph, having a simple quartz lens for its camera objective, since it gives directly the necessary focus and inclination of the plates for every desired angle between collimator and camera.

II. RELATIVE LIGHT-POWER OF SIMPLE AND COMPOUND PRISMS.

Various means are available for giving a definite dispersion to a parallel beam of given section, such as emerges from the

objective of the collimator of the spectroscope. We may employ systems of simple prisms of either large or small refracting angle, different varieties of compound prisms, and finally diffraction gratings. Of all these dispersion systems that one should be selected which involves the least loss of light while yielding the dispersion sought, the other conditions prescribed by the particular purpose of the spectroscope being also taken into consideration.

As far as simple prisms are concerned, it was shown by Pickering[1] as early as 1868 that the most favorable form of the prism is that at which the incident beam is completely polarized. The corresponding angle of the prism will vary according to the index of refraction of the ray which is to pass through the prisms at the angle of minimum deviation. The following values are obtained:

Index	Angle of prism
1.50	67°22′.8
1.55	65 39.4
1.60	64 0.6
1.65	62 26.2
1.70	60 55.8

The most favorable angle for the prism, therefore, lies between 60° and 67° for the ordinary kinds of glass, and a prism of 60° angle may be regarded as a very suitable one in all cases. Pickering's beautiful investigation does not appear to have been correctly understood by all, as the assertion is still met with in many treatises that the greatest possible refracting angle is always advantageous in respect to light-power where simple prisms are employed.

Another investigation[2] of the brightness of the spectrum produced by simple prisms has been given by Krüss. He gives the proof that the adjustment of the prisms for the minimum of deviation, which is so important for the purity of the spectrum, is also most favorable for its brightness.

[1] "On the comparative efficiency of different forms of the spectroscope." *Am. Jour.* (2) **45**, 301, 1868.

[2] "Ueber Spektralapparate mit automatischer Einstellung" *Zeitschrift fur Instrumentenkunde*, **5**, 181, 1885.

No thorough investigations on the loss of light in compound prisms[1] seem to have been made, but it is generally assumed that they are decidedly superior in light-power to simple prisms. The reason assigned for this is that a compound prism, in which the ray has to pass but twice between air and glass, can be so constructed as to give the same dispersion as two simple 60° prisms, in which a large loss of light by reflection occurs four times at the boundary of air and glass. The gain in light accomplished in this way is overestimated, however, and the calculation shows that this gain may be entirely cancelled by the loss of light due to the surrender of the angle of incidence which is specially favorable for a 60° prism, as well as by the loss, especially for the rays of shorter wave-length, caused by the large increase in the thickness of the glass.

While it is possible in the case of a simple prism to express the relation between the brightness of the spectrum, the dispersion, and the refracting angle by a not too seriously complicated formula, the corresponding expression in the case of the compound prism becomes entirely confused. It is not possible to express by a single formula the intensity of spectrum given by a compound prism of any given kind of glass, with given refracting angle and given dispersion, in such a way that we could be in position to find the most favorable conditions for brightness by a discussion of such a formula. I have, therefore, calculated rigorously the dispersion and light-power for a number of different kinds of compound prisms, as well as for systems of simple prisms, for comparison; and I communicate the numerical results below, as they may serve as a basis for others elsewhere, wherever the kind of glass employed does not differ too strongly from that on which my calculation is based.

I first summarize all the necessary formulæ, the derivation of which is so simple that I need not give it here.

[1] It may be remarked in passing that the widely adopted designation of the ordinary compound prism, in which a prism of strong dispersion with a large refracting angle is placed between two acute angled prisms of slight dispersion, as "Rutherfurd prisms" lacks a historical basis. Browning is responsible for the idea and the first actual construction of such prisms. Rutherfurd suggested a less useful quintuple prism. *Monthly Notices*, 31, 203, 1871.

Let us call

Q the diameter of the incident beam,
J the angle of incidence,
R the angle of refraction,
A the deviation of the ray,
D the dispersion for a difference of wave-length of $1\mu\mu$,
S the length of face of the prism,
B the length of base of the prism,
c the coefficient of absorption of the glass for 1 mm thickness.
n the index of refraction for the ray passing at the minimum of deviation,
dn, dJ, dR, the variations of the corresponding quantities for a difference of wave-length of $1\mu\mu$,
b the refracting angle for the simple prism,
i the refracting angle of flint glass for the compound prism,
a the refracting angle of the crown glass for the compound prism,
m the number of the prisms.

For compound prisms all the data which refer to the exterior (crown) prisms are given the subscript a, those referring to the interior prisms the subscript i, while J and R receive the subscripts 1 and 2 for the simple prism, and 1 to 4 for the compound prism for the passage through the separate surfaces.

FORMULÆ FOR THE SIMPLE PRISM.

Path of the rays: $$\left(J_2 = R_1,\ R_2 = J_1\right)$$

$$R_1 = \frac{b}{2}$$

$$\sin J_1 = n \sin R_1$$

Dimensions: $$S = \frac{Q}{\cos J_1}$$

$$B = 2S \sin \frac{b}{2}$$

Deviation: $$A = 2J_1 - b$$

Dispersion: $$D = dR_2 = \frac{2}{n} \tan J_1\, dn - dJ_2$$

Light power: $$H = \frac{c^B - 1}{B \log c} (H_1 + H_2) ,$$

where $$H_1 = \frac{1}{2} \left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2 (J_1 + R_1)} \right)^2$$

and $$H_2 = \frac{1}{2} \left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2 (J_1 + R_1) \cos^2 (J_1 - R_1)} \right)^2 .$$

H_1 and H_2 are the intensities of the light polarized perpendicular and parallel to the refracting edge, the intensity of the incident light being called unity.[1]

FORMULÆ FOR THE COMPOUND PRISM.

Path of the rays: $(J_3 = R_2 , \ R_3 = J_2 ; \ J_4 = R_1 , \ R_4 = J_1)$

$$R_2 = \frac{i}{2}$$

$$\sin J_2 = \frac{n_i}{n_a} \sin R_2$$

$$R_1 = J_2 - a$$

$$\sin J_1 = n_a \sin R_1$$

Dimensions: $$S_a = \frac{Q}{\cos J_1}$$

$$B_a = \frac{S_a}{\cos J_2} \sin a$$

$$S_i = \frac{S_a}{\cos J_2} \cos R_1$$

$$B_i = 2 S_i \sin \frac{i}{2}$$

Deviation: $$A = 2 (J_1 - R_1 + J_2 - R_2)$$

Dispersion: $$D = dR_4 = \frac{2 \cos R_1 \sin R_2}{\cos J_1 \cos J_2} dn_i - \frac{2 \sin a}{\cos J_1 \cos J_2} dn_a - dJ_1$$

Light power: $$H = \left(\frac{C_a - 1}{B_a \log c_a} \right)^{B_a} \left(\frac{C_i - 1}{B_i \log c_i} \right)^{B_i} (H_1 + H_2) ,$$

where $$H_1 = \frac{1}{2} \left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2 (J_1 + R_1)} \right)^2 \left(\frac{\sin 2J_2 \sin 2R_2}{\sin^2 (J_2 + R_2)} \right)^2$$

and

$$H_2 = \frac{1}{2} \left(\frac{\sin 2J_1 \sin 2R_1}{\sin^2 (J_1 + R_1) \cos^2 (J_1 - R_1)} \right)^2 \left(\frac{\sin 2J_2 \sin 2R_2}{\sin^2 (J_2 + R_2) \cos^2 (J_2 - R_2)} \right)^2 .$$

[1] The derivation of these well-known formulæ of H_1 and H_2 may be found, for instance, in Winkelman's *Handbuch der Physik*, Bd. 2, Abth. I, 751.

My computations refer to the glasses from which the compound prisms were made for the spectrograph described in the *Publicationen des Astrophysikalischen Observatorium*, 7; it should be especially noted that the flint glass is very transparent, and is very slightly colored. These two glasses are from the works of Schott & Co., in Jena, and are designated in their price list as 0.102 (heavy silicate flint), and 0.144 (boro-silicate crown). In order to determine the differential quotients of the indices of refraction $\frac{dn}{d\lambda}$, the following values of the indices for three rays were taken from the price list mentioned (July 1886):

	0.102	0.144
D	1.6489	1.5100
F	1.6626	1.5156
H	1.6744	1.5201

From these figures the following dispersion formulæ were derived for the two kinds of glass:

$$\text{For } 0.102,\ n = \frac{1.6122 + 13.91}{\lambda - 210.2}$$

$$\text{For } 0.144,\ n = \frac{1.4934 + 6.85}{\lambda - 176.5},$$

from which follow the values for $\lambda = 4340$:

$$\text{For } 0.102,\ dn = -0.000277\ d\lambda$$

$$\text{For } 0.144,\ dn = -0.000103\ d\lambda.$$

I. SIMPLE PRISMS.

b	*S*	*B*	*m*	*A*	*D*	log *H*
30°	38.8mm	20.1mm	1	21°21′.8	32′.8	9.9148
			2	42 43.6	65.6	9.8302
			3	64 5.4	98.5	9.7461
			4	85 27.2	131.3	9.6625
			5	106 49.0	164.1	9.5795
45	45.6	34.9	1	34 41.9	57.0	9.8906
			2	69 23.8	113.9	9.7849
			3	104 5.7	170.9	9.6829
			4	138 47.6	227.8	9.5844
			5	173 29.5	284.8	9.4891

I. SIMPLE PRISMS—*Continued.*

b	S	B	m	A	D	log H
55	55.2	51.0	1	46 16.6	83.2	9.8578
			2	92 33.2	166.4	9.7274
			3	138 49.8	249.6	9.6079
			4	186 6.4	332.8	9.4980
			5	231 23.0	416.0	9.3961
60	64.0	64.0	1	53 41.5	104.5	9.8262
			2	107 23.0	208.9	9.6737
			3	161 4.5	313.4	9.5397
			4	214 46.0	417.9	9.4203
			5	268 27.5	522.4	9.3117
63	72.3	75.5	1	59 3.7	123.3	9.7952
			2	118 7.4	246.6	9.6213
			3	177 11.1	369.8	9.4723
			4	236 14.8	493.1	9.3414
			5	395 18.5	616.4	9.2226
65	80.2	86.2	1	63 13.7	140.6	9.7647
			2	126 27.4	281.3	9.5694
			3	189 41.1	421.9	9.4046
			4	252 54.8	562.6	9.2598
			5	316 8.5	703.2	9.1272

II. COMPOUND PRISMS.

i	a	S_i	B_i	S_a	B_a	m	A	D	log H
80°	8°	98.8mm	127.0mm	87.4mm	17.2mm	1	68°48′.8	186′.4	9.6908
						2	137 37.6	372.9	9.4132
	10	83.3	107.1	71.9	17.7	1	61 44.6	153.4	9.7488
						2	123 29.1	306.8	9.5172
	12	74.4	95.6	62.7	18.5	1	56 6.7	133.7	9.7797
						2	112 13.5	267.4	9.5720
	15	66.2	85.1	54.0	19.8	1	49 14.5	114.9	9.8047
						2	98 29.1	229.8	9.6162
	20	58.7	75.5	45.8	22.2	1	40 12.9	96.3	9.8228
						2	80 25.9	192.5	9.6481
90	15	101.9	144.1	79.2	32.7	1	67 30.7	195.6	9.6666
						2	135 1.4	391.2	9.3591
	20	77.4	109.4	56.7	30.9	1	53 43.6	141.0	9.7538
						2	107 27.2	282.0	9.5165
	25	67.5	95.4	47.2	31.8	1	44 9.7	117.2	9.7815
						2	88 19.5	234.5	9.5665
100	20	137.2	210.3	92.9	59.2	1	75 43.7	271.4	9.5196
						2	151 27.4	552.7	9.0770
	25	95.5	146.3	60.8	47.9	1	59 42.9	180.8	9.6758
						2	119 25.8	361.5	9.3642

The coefficients of absorption were found by taking the means of the measures by H. C. Vogel, Müller and Wilsing,[1] whose observations were made on varieties of glass almost identical, but not the same as the glass of the above prisms. For light of the wave-length 4340 I have taken the absorption coefficient for a thickness of 100 mm as 0.53 and 0.72 respectively. Hence we get for 1 mm thickness of glass

For 0.102, log c = 9.9972428,
For 0.144, log c = 9.9985733.

With these optical constants the computations were made for the following prism systems, for an incident beam of 35 mm section, and with the ray $H\gamma$ at the minimum deviation. The simple prisms are computed for the flint glass 0.102.

I have graphically represented the values of D and log H in Fig. 2, in order to give a clear survey of the relation between the dispersion and the light-power of the different systems of prisms. The curves for systems of simple prisms are given by the continuous line, for compound prisms by the broken line. The appended numbers are the refracting angles in degrees.

We can see at a glance that, aside from the very slight dispersion given by only a single prism of angle less than 60°, the simple prism with a refracting angle of from about 59° to 64° gives the greatest intensity for every dispersion, inasmuch as the points of their curves always lie highest. For instance, if a dispersion of 250″ is desired for $d\lambda = 1\,\mu\mu$ in the region of $H\gamma$, this can be obtained in the following ways:

Two	simple	prisms	of	about	63°	angle	give	an	intensity	of 0.413
Three	"	"	"	"	55°	"	"	"	"	" 0.405
Two	compound	"	"	"	80°	and 13°	"	"	"	" 0.391
Four	simple	"	"	"	47°		"	"	"	" 0,368
One	compound	"	"	"	121°		"	"	"	" 0.362
Two	"	"	"	"	90°	and 23°	"	"	"	" 0.355
Five	simple	"	"	"	41°		"	"	"	" 0.327

In a similar way the appropriate system of prisms and their light-power may be read off from the curve for every other

[1] This JOURNAL, 5, 75, 1897.

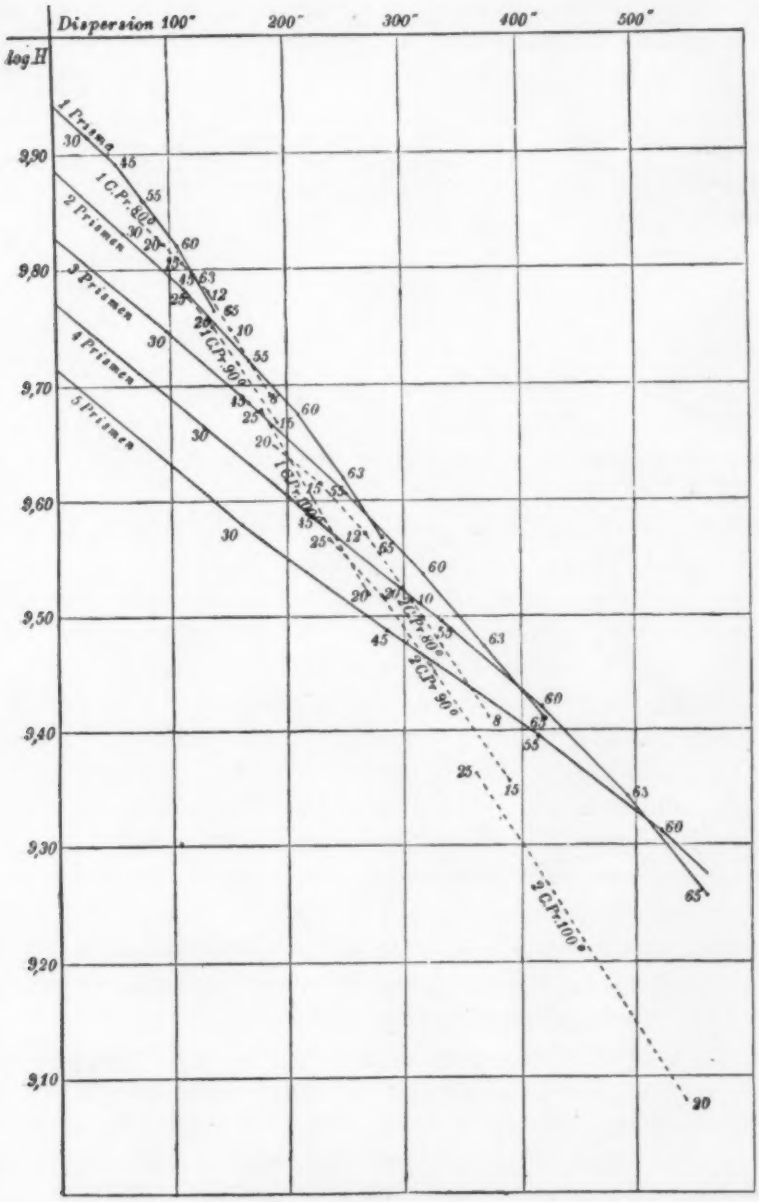

Dispersion 100" 200" 300" 400" 500"
log H
2,90
2,80
2,70
2,60
2,50
2,40
2,30
2,20
2,10
1 Prisma
1 C.Pr.800
2 Prismen
3 Prismen
4 Prismen
5 Prismen
2 C.Pr.900
1 C.Pr.900
2 C.Pr.800
2 C.Pr.1000

Fig. 2.

dispersion. I must especially emphasize here the fact that compound prisms, although they may equal the light-power of simple prisms for faint dispersion become increasingly unsuitable for very high dispersions. Thus, if we draw a sort of common tangent to all the curves of the simple prism, this will come out distinctly less steep than the corresponding line for the compound prism. This is related to the fact that the light is completely polarized in the most favorable form of simple prisms, so that more than the half of its intensity can never be lost by the reflections. If a system of such prisms has almost completely polarized the light, the loss of light by reflection on increasing the number of prisms is almost zero. We cannot avail ourselves of this favorable circumstance in the case of compound prisms, since with them there are always two different angles of incidence.

Although it might be possible that the light-power of compound prisms should turn out a little better for some other point in the spectrum, it is clearly evident from the above that compound prisms are by no means superior in brightness to simple prisms in the region of the most active photographic rays. It will be necessary to prove by computation, ultimately with the employment of different kinds of glass, whether in a given case the compound prism does bring an increase of light, which certainly cannot be large. If we further consider that variations of temperature must always produce tensions of the hard-cemented compound prisms which affect the sharpness of the image, and further that the surface errors have a greater effect at the large angles of incidence which occur at the flint glass of the compound prisms, we shall conclude that such prisms can by no means be employed to advantage in the construction of spectrographs for faint objects, especially when the apparatus must be used in very different temperatures, as is the case for stellar spectrographs. Since very heavy and strongly yellow kinds of flint glass are not infrequently used for increasing the dispersion of compound prisms, I will remark here that such prisms are not at all permissible for spectrographs, since flint glass of that sort

completely cuts off a large part of the photographically active rays.

As a result of the above investigations there can be no doubt that only simple flint prisms of about 60° angle could come in question in the construction of the new stellar spectrographs for the Astrophysical Observatory. Since the dispersion of the former spectrograph for the determination of stellar motions in the line of sight (397.4 for 1$\mu\mu$) had proven very satisfactory, a similar dispersion was planned for the larger of the new instruments, which is designated as spectrograph No. III. The graphical representation now shows at once that this condition could be met in two ways with an almost exactly equal intensity of light—either by three prisms of about 64° or by four prisms of about 59° angle. For the former the deviation of the ray would amount to about 184°, for the latter 208°, and since a very simple form of construction can be made for 180° deviation, a system of three prisms of the flint glass 0.102 was selected for spectrograph No. III. The refracting angle was now so computed that the ray $H\gamma$ suffers a deviation of exactly 180°; this condition required an angle of 63° 28′, and the firm of C. A. Steinheil's Sons accomplished the problem of giving the three prisms this angle with a highly praiseworthy accuracy. The investigation of the prisms yielded the following values for the refracting angles:

Prism	No. 1	63° 28′	17.″13
"	No. 2	63 26	55.47
"	No. 3	63 27	26.37

The index of refraction for $H\gamma$ has precisely the same value for the three prisms, 1.67435, so that when placed at the minimum of deviation for $H\gamma$ the light-power of the system would reach its maximum at a refracting angle of 61° 42′. We see that in this spectrograph this most favorable angle is very nearly taken, and the slight difference in that value is entirely without effect.

[*To be continued.*]

THE VELOCITY IN THE LINE OF SIGHT OF ϵ *LEONIS.*

By W. H. Wright.

In this Journal, **11**, 307, Mr. W. S. Adams announced the variable velocity of ϵ *Leonis* (1900, $\alpha = 9^h\ 40^m.2$, $\delta = +24^\circ\ 14'.1$). The announcement was made on the basis of fifteen spectrograms secured with a spectroscope attached to the forty-inch refractor of the Yerkes Observatory.

Below are given the results of the measurements of all the spectrograms of this star obtained with the Mills spectrograph. At the time of receiving Mr. Adams' article the first five had been secured. As they showed no range which might not be due to accidental errors of observation, the remaining three observations were made for the purpose of confirming the variation.

		Gr. M. T.	Velocity
1897	April 8	$16^h.3$	+5 [1]
1898	Jan. 26	20 .4	+4.4
1898	Feb. 3	20 .2	+4.2
1898	Feb. 8	20 .1	+6.9
1900	Jan. 16	23 .8	+5.4
1900	May 24	16 .8	+5.0
1900	May 28	17 .0	+4.8
1900	May 30	16 .8	+4.9

It will be seen that the measurements give no evidence of variable velocity. The agreement of the determinations cannot be due to a purely fortuitous distribution of the observations between two points on the velocity curve having equal velocity, as will be seen by comparing the intervals between epochs of

[1] Rough measurements of a poor plate, not to be used in the final discussion of the star's motion.

observation with the period which Mr. Adams assigns—two and a quarter days.

LICK OBSERVATORY.
UNIVERSITY OF CALIFORNIA,
June 2, 1900.

NOTE.—In the absence of Mr. Adams, who made the observations and measurements of ε *Leonis* during the usual winter vacation of the undersigned, it seems proper that a statement should be given as to the accordance of measures made by Mr. Adams upon spectrum plates of other stars considered to be of uniform radial velocity. As far as known, the instrumental conditions were precisely the same as for the spectrograms of ε *Leonis*. The dates are those on which the spectrograms were taken.

	γ *Andromedae*			α *Canis Minoris*	
1899	Sept. 27	−13.8 km	1899	Nov. 3	−6.9 km
	Nov. 28	−15.1		Nov. 28	−4.2
	Dec. 7	−14.6		γ *Leonis*	
	Dec. 23	−16.9	1899	Dec. 7	−39.2
	α *Arietis*			Dec. 23	−40.3
1899	Oct. 6	−20.5		α *Lyrae*	
	Dec. 23	−20.7	1899	Oct. 18	−19.6
	ε *Aurigae*			Oct. 18	−18.1
1899	Nov. 28	−0.2		α *Ursae Majoris*	
	Dec. 6	+0.1	1899	Dec. 7	−11.5
	Dec. 7	+0.3	1900	Jan. 28	− 9.8

EDWIN B. FROST.

ERRATA.

VOL. XI, April 1900. The formula $m_p = [\bar{7}.20386]PV^3$, which appears in the text on p. 249, should terminate the second footnote on that page. On p. 250, line 8, of Addendum, *for* "$m = 6$" *read* "$m > 6$."

Minor Contributions and Notes

MEASUREMENT OF PHOTOGRAPHIC INTENSITIES.[1]

The comparison of various celestial and terrestrial sources of light has for many years been a subject for investigation by the officers of this Observatory. One of the first comparisons of the light of the Sun and Moon was made by Professor George P. Bond in 1861 (*Mem. Amer. Acad.* **8**, N. S., 298). Comparisons of the light of the solar corona, of the Moon, and of twilight, with a standard candle were made by the writer in 1870. A comparison of various artificial lights was made by Professor W. H. Pickering, as his graduating thesis in 1879 (*Proc. Amer. Acad.* **15**, 236). Since 1887, all the photographs obtained with the principal instruments of this Observatory have had an image of a standard light photographed on them for purposes of comparison. In 1887, when Mr. W. H. Pickering was placed in charge of the Boyden Department, an important part of his work was a quantitative determination of photographic intensities. This investigation was completed in 1891, and the results are published in the *Annals*, **32**, chapter I. The relative brightness of 10,498 stars, for the wavelength 430, was determined by Mrs. Fleming and was published in 1890 in the Draper Catalogue (*Annals*, **26** and **27**.) An independent determination of the total photographic brightness of the stars was undertaken in 1896 by photographing stars out of focus. As none of these investigations served to determine certain constants, which seemed to the writer to be important, a more extensive investigation was assigned to Mr. Edward S. King, under whose direction the photographs are taken at Cambridge. This work has been modified and extended by Mr. King, so that it now includes the following researches, which he describes below:

In 1896, a monthly test of plates was inaugurated, which was of a routine character. A complete determination, not only of the sensitiveness of the emulsion, but of the constancy of the light and of the developer employed, is obtained by the expenditure of one 8×10 plate per month (*Photogram*, **6**, 261). The means of the results of

[1] *Harvard College Observatory Circular* No. 50.

these tests, extending over a long period of time and necessarily subject to all the accidental errors due to season or other factors, will probably show the effect of the age of the plate on its sensitiveness, whether the image is affected by the interval between the exposure and the development, and other facts of similar interest.

About the same time with the above, the Director suggested that an extensive investigation be undertaken of the photographic measurement of various sources of light. The work was to be systematized into a routine, which should occupy the least time in making the observations. By extending the period over years and by performing each experiment under the most diverse circumstances, the final results should be of great accuracy. Preliminary to this work, an apparatus was constructed for making photographic wedges in which equal intervals of the scale corresponded to equal differences in stellar magnitude. The general design was the same as that described in the *Annals*, **26**, 6, except that the triangular aperture was replaced by an aperture bounded by a logarithmic curve. The part of the curve extending to minus infinity, but enclosing a finite area, is represented by admitting light through an additional aperture of equivalent area. This apparatus having a range in aperture of five magnitudes, is useful for measuring the intensity of surfaces directly, or of bright objects, when the aperture is covered with porcelain. It is also of value in comparing the sensitiveness of plates, and for studying the relation of the darkening of the film to the exposure, and to the intensity of the light.

The object of this investigation is to determine the photographic intensity of various sources of light upon a uniform scale. This scale will be that of the Meridian Photometer, in which α *Ursae Minoris* has the magnitude of 2.15, and one unit corresponds to the ratio of 2.518, whose logarithm is 0.400. Luminous points may be compared directly with stars. In the case of surfaces, the light emitted by a circle having a diameter of one minute of arc is employed. In general, different portions of the same photographic plate are exposed for a given time to the sources of light to be compared, and the darkening measured with a photographic wedge.

Observations of surfaces will include, intensity of the sky at different distances from the Sun; the sky in the zenith during twilight, on clear and cloudy days, on dark and moonlight nights; comparisons of blue sky with cumulus cloud; intensity of Milky Way, Aurora, and

Zodiacal Light. For this purpose pinhole cameras are used with various apertures. For very faint lights, apertures subtending an angle of 60° or more are used. When possible, automatic devices are employed, both to make the exposure and to shift the plate.

For measuring luminous points, the light concentrated by a lens illuminates the plate placed at various distances from the focus. In this manner, bright stars, the planets, and the Moon at different phases and altitudes, may be compared with *α Ursae Minoris* or an artificial light placed at a distance. It is also necessary to determine the effect of any slight fogging before, after, or during the principal exposure. The total and local absorption of the lens will also be measured. The same objects are also compared directly by pinhole cameras. In using a pinhole camera to measure points differing greatly in brightness the duration of the exposure must be varied. This necessitates a study of the darkening of the plate with relation to time. The comparison is made, in all cases, with a light, producing an equal darkening in the same time. Another factor may be introduced by inclining the plate. An angle of 60° is equal to a diminution of one half, or about three quarters of a magnitude. The exact amount corresponding to the inclination used, must be determined by experiment.

The determination of the light of the Sun is a difficult problem. The light may be reduced by a distant lens of short focus, and compared with the standard light similarly reduced, but at less distance. Another way is by a combination of lenses like a telescope, which enlarges or reduces objects, as we look through one end or the other. The absorption of the instrument is thus eliminated. Two plates of glass, placed so as to give multiple reflections, will also afford a wide angle of exposure, as well as a means of comparing lights of great difference in intensity. Pinhole cameras having the aperture covered with a porcelain plate can be constructed so as to give a range of 15 to 20 magnitudes, which is sufficient for comparing the Sun and the Moon. A second pinhole camera placed in front will cut off the light of the sky from the aperture. The absorption and the diffusing power of the porcelain plate must be measured independently. In all the preceding plans the sensitiveness of the plates to different colors affects the results. By combining a slit spectroscope with the cameras just described, a comparison of the light of the Sun and the Moon may be made in different parts of the spectrum, corresponding to given wavelengths.

The comparison of the spectrum of the various sources of light possesses many advantages. The work is placed on such a basis that the results are freed from the troublesome questions of absorption, sensitiveness of the plate, etc., and are rendered directly comparable with those that may be obtained at different times, by other observers, under widely differing circumstances. In fact, the results should be the same as those obtained by the eye, the bolometer, or in any other manner that may be devised. All these measures may be made either with a telescope and objective prism, or with a slit spectroscope combined with a pinhole camera or telescope. With the objective prism the spectra are made of the same width, either by interposing a cylindrical lens, by moving the plate at the same equable rate for each exposure, or by throwing the image out of focus. In the latter case, a rectangular aperture should be placed over the prism, so as to give width with a minimum loss of definition. With the moving plate, a variation of light is obtaind by covering one half of the prism at a time, along a line perpendicular to its edge.

The standard light used is an ordinary Argand gas burner shining through a small hole. The star α *Ursae Minoris* is made the ultimate standard to which all work is referred, since the standard light is compared with it before and after each monthly test of the plates. In addition to the wedges for measuring the density of photographic plates a polarizing photometer is used for comparing surfaces with each other.

Early in 1900, while this work was in progress, Mr. W. H. Pickering, in preparing to observe the Solar Eclipse of May 28, desired to select a suitable plate and developer for the work, and accordingly undertook the following investigation, which he describes below:

A suitable standard of light has long been wanted in photography. Artificial sources usually give even more uncertain results photographically than they do visually, because a slight variation in temperature will effect the blue end of the spectrum even more than the red end. In 1893 it occurred to the writer to employ as a primary standard of actinic intensity the radiation of a star shining directly upon the photographic plate, without previously passing through or being reflected from any medium, except our atmosphere. Since this standard is too faint for general use, a secondary one to be standardized from it has been devised. The light from the star is condensed through a simple plano-convex lens of 8.2 cm aperture and short focus, and is focused on a small circle of ground glass 0.5 cm in diameter. This is placed 3 cm in front of the sensitive plate, which is exposed to it through a small square aperture measuring 0.2 cm on a

side, cut in a blackened brass plate. The constant of this instrument was determined, and it was found to give about thirty times the light of the direct radition of a star upon the photographic plate. With twenty minutes exposure α *Ursae Minoris* darkened the plate sufficiently to produce the "Sensitive Tint," that is to say the tint where a small variation in the light is most noticeable. The secondary standard, however, is not sufficiently brilliant for ordinary purposes, and a tertiary one has therefore been devised. This consists of a box 30 cm in length, one end of which contains an aperture 5 cm in diameter, covered by a piece of ground glass, and the other end carries the sensitive plate and the blackened brass plate described above. Just inside the ground glass may be placed diaphragms ranging from 0.04 to 4.0 cm in diameter. At a distance of about 200 cm beyond the ground glass is placed an Argand gas burner. This apparatus gives ample light for all photographic purposes, and the plates exposed in it may always be standardized when necessary by an additional exposure to the secondary standard.

The three pieces of apparatus above described are adapted to various investigations. Thus the photographic brightness of the Moon, and of the brighter stars and planets, may be measured with accuracy by employing different apertures in front of the lens of the secondary standard, and measuring the brightness of the various images obtained by means of a photographic wedge, or a series of standard squares of varying density. It is very important to vary the aperture rather than the time of exposure, since the results obtained in the latter case would have to be corrected by the "Time Correction" (*Annals*, **32**, 20). The secondary standard also enables us to express the sensitiveness of a plate, in terms of universal application throughout all time. Thus the Seed plate No. 27 is capable of being appreciably darkened when exposed for ten minutes to a source equal to 300 times the brightness of α *Ursae Minoris*. This amount of light is equal to about thirty times the brightness of a star whose photographic magnitude is 0.0. The logarithm of this number, 1.5, may be conveniently used to represent the sensitiveness of the Seed plate. By the use of the secondary and tertiary standard, together with an ordinary photographic telescope, we may make a study of the brightness of nebulæ, comets and other luminous surfaces.

From these various investigations it is hoped that we shall obtain a scale of photographic intensities with which all sources of light may be compared and to which they may be referred.

EDWARD C. PICKERING.

May 9, 1900.

INDEX TO VOLUME XI.

SUBJECTS.

PAGE

ALGOL Stars, Density of. *H. Seeliger* - - - - - - - 247
Variable, Mme. Ceraski's Second. *Edward C. Pickering* - - 165
AMERICAN Eclipse Parties, Plans of - - - - - - - 314
ARC-SPECTRA of Metals, Researches on. *B. Hasselberg* - - - 67
ATMOSPHERES, Escape of Gases from Planetary. *S. R. Cook* - - 36
G. Johnstone Stoney 251, 357
BALANCING Rheostat for Bolometers. *P. G. Nutting* - - - - 44
BINARY χ *Draconis*, Orbit of Spectroscopic. *W. H. Wright* - - 131
BLACK Body, Distribution of Energy in Spectrum of. *F. Paschen* - 288
BOLOMETERS, Balancing Rheostat for. *P. G. Nutting* - - - 44
BROMIDE of Silver Gelatine, Deviation from Law of Reciprocity for. *K. Schwarzschild* - - - - - - - - - - - 89
Effect of Intermittent Exposure on. *K. Schwarzschild* - - 92
BRUCE, CATHERINE WOLFE - - - - - - - - - 168
CALIBRATION of the Slit in Spectral Photometric Measurements. *E. V. Capps* - - - - - - - - - - - 25
Capella, System of. *J. Miller Barr* - - - - - - - 248
CERASKI'S, Mme., Second *Algol* Variable. *Edward C. Pickering* - 165
CHROMOSPHERE, Lines Observed in Spectra of Stars and of. *W. N. Hartley* - - - - - - - - - - - - 163
COMPARISON Spectra, Lines of Titanium for. *Lewis E. Jewell* - 243
CROSSLEY Reflector of the Lick Observatory. *James E. Keeler* - 325
β *Crucis* TYPE, Lines in the Spectra of Stars of. *Joseph Lunt* - 262
CURVE, Luminosity, of the Solar Spectrum. *D. W. Murphy* - - 220
CURVATURE of Spectral Lines in Spectroheliograph. *W. S. Adams* - 309
Y Cygni Elliptic Elements of. *N. C. Dunér* - - - - - - 175
DENSITY of *Algol* Stars. *H. Seeliger* - - - - - - - 247
DISPERSION, Prism of Uniform. *Charles G. Abbot* and *Frederick E. Fowle, Jr.*, - - - - - - - - - - - - - 135
DISTRIBUTION of Energy in Spectrum of Black Body. *F. Paschen* - 288
DIURNAL Rotation of Optical Field of Siderostat and Heliostat. *M. A. Cornu* - - - - - - - - - - - 148
χ *Draconis*, Orbit of Spectroscopic Binary. *W. H. Wright* - - 131
DYNAMICS, Attempt to Test the Nebular Hypothesis by an Appeal to the Laws of. *F. R. Moulton* - - - - - - - - - 103

PAGE

ECLIPSE, Indian, Spectrographic Results Obtained at the. *W. W. Campbell* - - - - - - - - - - - - 226
Parties, American, Plans of - - - - - - - - - 314
Problems, Solar. *George E. Hale* - - - - - - - 47
Solar, Photometric Observation of *Mercury* during. *G. Müller* - 144
ELEMENTS, Elliptic, of the System of *Y Cygni*. *N. C. Dunér* - 175
ENERGY, Distribution of, in Spectrum of Black Body. *F. Paschen* - 288
Eros, Opposition of, in 1900. *Edward C. Pickering* - - - - - 244
ERROR, Probable, of a Radial Velocity Determination. *W. W. Campbell* - - - - - - - - - - - - - 167
ESCAPE of Gases from Planetary Atmospheres. *S. R. Cook* - - 36
G. Johnstone Stoney 251, 357
EXPOSURE, Intermittent, Effect on Bromide of Silver Gelatine. *K. Schwarzschild* - - - - - - - - - - - 92
FIELD of Siderostat and Heliostat, Diurnal Rotation of. *M. A. Cornu* 148
GASES, Escape of, from Planetary Atmospheres. *S. R. Cook* - - 36
G. Johnstone Stoney 251, 357
HELIOSTAT, Diurnal Rotation of Field of. *M. A Cornu* - - - 148
β Herculis, Variable Velocity of. *W. W. Campbell* - - - - - 140
HERSCHEL, SIR JOHN, Magnitudes of 919 Stars Observed by. *W. Doberck* - - - - - - - - - - - 192–270
H. I. 55 Pegasi, Spiral Nebula. *James E. Keeler* - - - - - 1
HYDROGEN, Spectrum of. *Victor Schumann* - - - - - - 312
INTENSITIES, Measurements of Photographic. *Edward C. Pickering* 416
INTERMITTENT Exposure on Bromide of Silver Gelatine, Effect of. *K. Schwarzschild* - - - - - - - - - - - 92
INTERMERCURIAL Planet, Photographic Search for. *Edward C. Pickering* - - - - - - - - - - - - 322
KINETIC Theory, Escape of Gases from Planetary Atmospheres according to. *S. R. Cook* - - - - - - - - - - 36
G. Johnstone Stoney - - - - - - - - 251, 357
LICK Observatory-Crocker Expedition, Spectrographic Results Obtained at the Indian Eclipse by. *W. W. Campbell* - - - - - 226
LICK Observatory, Crossley Reflector of the. *James E. Keeler* - - 325
ε Leonis, Variable Velocity of. *W. S. Adams* - - - - - - 307
LINE of Sight, Variable Velocity of *β Herculis* in. *W. W. Campbell* 140
Variable Velocity of *ε Leonis* in. *W. S. Adams* - - - - - 307
W. H. Wright - - - - 414
Velocities in. *H. C. Vogel* - - - - - - - - - 373
LINES Observed in the Spectra of Stars and of the Chromosphere. *W. N. Hartley* - - - - - - - - - - - - 163
In Spectra of Stars of the *β Crucis* Type. *Joseph Lunt* - - 262
Of Titanium for Comparison Spectra and their Prominence in the Chromosphere. *Lewis E. Jewell* - - - - - - - - 243

PAGE

LUMINOSITY Curve of the Solar Spectrum. *D. W. Murphy* - - 220
MAGNITUDE, Star, Physical Meaning of. *R. de Kövesligethy* - - 350
MAGNITUDES of 919 Fixed Stars. *W. Doberck* - - - 192–270
Mercury, Photometric Observation of. *G. Müller* - - - - 144
METALS, Arc-Spectra of, Researches on. *B. Hasselberg* - - - 67
MILLS Spectrograph, Temperature Control of. *W. W. Campbell* - 259
MOON'S Theoretical Spectrographic Velocity. *W. W. Campbell* - 141
NEBULA, Spiral, *H. I. 55 Pegasi*. *James E. Keeler* - - - - 1
NEBULAR Hypothesis, Attempt to Test the. *F. R. Moulton* - - 103
OPPOSITION of *Eros* in 1900. *Edward C. Pickering* - - - - 244
ORBIT of the Spectroscopic Binary, χ *Draconis*. *W. H. Wright* - 131
PHOTOGRAPHIC Intensities, measurement of. *Edward C. Pickering* - 416
Search for an Intermercurial Planet. *Edward C. Pickering* - 322
Notes. *Lewis E. Jewell* - - - - - - - - - 240
PHOTOMETRIC Measurements, Calibration of Slit in Spectral. *E. V. Capps* - - - - - - - - - - - - - 25
Work, New System for Spectral. *D. B. Brace* - - - - - 6
Observation of *Mercury* During the Total Solar Eclipse on May 28, 1900. *G. Müller* - - - - - - - - 144
PHYSICAL Meaning of the Star-Magnitude. *R. de Kövesligethy* - 350
PLANETARY Atmospheres, Escape of Gases from. *S. R. Cook* - - 36
G. Johnstone Stoney 251, 357
PLANET, Intermercurial, Photographic Search for. *Edward C. Pickering* - - - - - - - - - - - - 322
Mercury, Photometric Observation of the. *G. Müller* - - 144
POTSDAM Astrophysical Observatory, Refractor of the - - - 100
Spectrographs. *H. C. Vogel* - - - - - - - - 393
PRISM of Uniform Dispersion. *Charles G. Abbot* and *Frederick E. Fowle, Jr.* - - - - - - - - - - - - 135
PROBABLE Error of a Radial Velocity Determination. *W. W. Campbell* - - - - - - - - - - - - 167
RADIAL Velocity Determination, Probable Error of. *W. W. Campbell* 167
RECIPROCITY, Deviations from Law of, for Bromide of Silver Gelatine. *K. Schwarzschild* - - - - - - - - - - - 89
RHEOSTAT, Balancing, for Bolometers. *P. G. Nutting* - - - 44
ROTATION, Diurnal, of Optical Field of Siderostat and Heliostat. *M. A. Cornu* - - - - - - - - - - - 148
SIDEROSTAT, Diurnal Rotation of Field of. *M. A. Cornu* - - 148
SILICON, Spectrum of. *Joseph Lunt* - - - - - - - 262
SLIT in Spectral Photometric Measurements, Calibration of. *E. V. Capps* - - - - - - - - - - - - - 25
SOLAR Eclipse on May 28, 1900, Photometric Observation of *Mercury* during. *G. Müller* - - - - - - - - - - 144

PAGE

Eclipse Problems. *George E. Hale* - - - - - - - 47
Spectrum, Luminosity Curve of. *D. W. Murphy* - - - 220
SPECTRA, Arc-, of Metals, Researches on. *B. Hasselberg* - - 67
Lines of Titanium for Comparison. *Lewis E. Jewell* - - 243
of Stars and of Chromosphere. *W. N. Hartley* - - - - 163
of Stars of the β *Crucis* Type, Lines in. *Joseph Lunt* - - 262
SPECTRAL Lines, Curvature of, in Spectroheliograph. *W. S. Adams* 309
Photometric Measurements, Calibration of Slit in. *E. V. Capps* 25
Photometric Work, New System for. *D. B. Brace* - - - 6
SPECTROGRAPH, Mills, Temperature Control of. *W. W. Campbell* - 259
SPECTROGRAPHS, Remarks on the Construction and Adjustment of. I. *J. Hartmann* - - - - - - - - - - - 400
New Potsdam. *H. C. Vogel* - - - - - - - - - 393
SPECTROGRAPHIC Results Obtained at the Indian Eclipse. *W. W. Campbell* - - - - - - - - - - - 226
Velocity, Moon's Theoretical. *W. W. Campbell* - - - 141
SPECTROHELIOGRAPH, Curvature of Spectral Lines in. *W. S. Adams* 309
SPECTROSCOPIC Binary, χ Draconis, Orbit of. *W. H. Wright* - - 131
Notes. *Lewis E. Jewell* - - - - - - - - - 234
SPECTRUM of Black Body, Distribution of Energy in. *F. Paschen* - 288
of Hydrogen Beyond λ 185μμ. *Victor Schumann* - - - 312
Luminosity Curve of Solar. *D. W. Murphy* - - - - 220
of Silicon. *Joseph Lunt* - - - - - - - - - 262
SPIRAL Nebula *H. I., 55 Pegasi. James E. Keeler* - - - - - 1
STAR-MAGNITUDE, Physical Meaning of. *R. de Kövesligethy* - - 350
STARS, Density of *Algol. H. Seeliger* - - - - - - - 247
of β *Crucis* Type, Lines in the Spectra of. *Joseph Lunt* - - 262
Magnitudes of 919 Fixed. *W. Doberck* - - - - 192–270
Spectra of, Lines Observed in. *W. N. Hartley* - - - - 163
SYSTEM of *Capella. J. Miller Barr* - - - - - - - 248
TEMPERATURE Control of the Mills Spectrograph. *W. W. Campbell* 259
TITANIUM, Lines of, for Comparison Spectra and their Prominence in the Chromosphere. *Lewis E. Jewell* - - - - - 243
VARIABLE, Mme. Ceraski's Second *Algol. Edward C. Pickering* - 165
Velocity of β *Herculis. W. W. Campbell* - - - - - 140
Velocity of ε *Leonis. W. S. Adams* - - - - - - 307
W. H. Wright - - - - - 414
VELOCITIES in the Line of Sight. *H. C. Vogel* - - - - 373
VELOCITY, Variable, of β *Herculis. W. W. Campbell* - - - 140
of ε *Leonis. W. S. Adams* - - - - - - - - 307
W. H. Wright - - - - - - - - 414
Determination, Probable Error of a Radial. *W. W. Campbell* - 167
Moon's Theoretical Spectrographic. *W. W. Campbell* - - 141

INDEX TO VOLUME XI.

AUTHORS.

PAGE

ABBOT, CHARLES G. and FREDERICK E. FOWLE, JR. A Prism of Uniform Dispersion - - - - - - - - - - 135

ADAMS, W. S. The Variable Velocity of ϵ *Leonis* in the Line of Sight - - - - - - - - - - - - 307

The Curvature of the Spectral Lines in the Spectroheliograph - 309

BARR, J. MILLER. The System of *Capella* - - - - - - 248

BRACE, D. B. On a New System for Spectral Photometric Work - 6

CAMPBELL, W. W. The Variable Velocity of β *Herculis* in the Line of Sight - - - - - - - - - - - - - 140

The Determination of the Moon's Theoretical Spectrographic Velocity - - - - - - - - - - - - - 141

On the Probable Error of a Radial Velocity Determination - 167

Some Spectrographic Results Obtained at the Indian Eclipse by the Lick Observatory-Crocker Expedition - - - - - 226

The Temperature Control of the Mills Spectrograph - - - 259

CAPPS, E. V. Calibration of the Slit in Spectral Photometric Measurements - - - - - - - - - - - - - 25

COOK, S. R. On the Escape of Gases from Planetary Atmospheres according to the Kinetic Theory - - - - - - - 36

CORNU, M. A. On the Law of Diurnal Rotation of the Optical Field of the Siderostat and Heliostat - - - - - - - 148

CREW, HENRY. Professor J. J. Thomson's Work on the Structure of the Atom - - - - - - - - - - - 170

DOBERCK, W. On the Magnitudes of 919 Fixed Stars Determined from Sequences Observed by Sir John Herschel during the years 1835 to 1838 - - - - - - - - - - 192–270

DUNÉR, N. C. Calculation of Elliptic Elements of the System of *Y Cygni* - - - - - - - - - - - - - 175

FOWLE, FREDERICK E. JR. and CHARLES G. ABBOT. A Prism of Uniform Dispersion - - - - - - - - - - 135

HALE, GEORGE E. Solar Eclipse Problems - - - - - - 47

Plans of American Eclipse Parties - - - - - - - 314

HARTLEY, W. N. On the Probable Origin of Some of the Lines Observed in the Spectra of Stars and of the Chromosphere - 163

PAGE

HARTMANN, J. Remarks on the Construction and Adjustment of Spectrographs, I - - - - - - - - - - - - 400
HASSELBERG, B. Researches on the Arc-Spectra of the Metals - 67
JEWELL, LEWIS E. Spectroscopic Notes - - - - - - - 234
Photographic Notes - - - - - - - - - - - 240
The Use of the Lines of Titanium for Comparison Spectra and their Prominence in the Chromosphere - - - - - - 243
KEELER, JAMES E. The Spiral Nebula *H. I, 55 Pegasi* - - - 1
The Crossley Reflector of the Lick Observatory - - - 325
KÖVESLIGETHY, R. DE. The Physical Meaning of the Star-Magnitude - - - - - - - - - - - - - - 350
LUNT, JOSEPH. On the Origin of Certain Unknown Lines in the Spectra of Stars of the β *Crucis* Type, and on the Spectrum of Silicon - - - - - - - - - - - - - - 262
MOULTON, F. R. An Attempt to Test the Nebular Hypothesis by an Appeal to the Laws of Dynamics - - - - - - - 103
MÜLLER, G. A Proposal for the Photometric Observation of the Planet *Mercury* during the Total Solar Eclipse on May 28, 1900 - - - - - - - - - - - - - 144
MURPHY, D. W. A Method of Determining the Luminosity Curve of the Solar Spectrum - - - - - - - - - - 220
NUTTING, P. G. On a Balancing Rheostat for Bolometers - - 44
PASCHEN, F. On the Distribution of Energy in the Spectrum of the Black Body at High Temperatures - - - - - - 288
PICKERING, EDWARD C. Mme. Ceraski's Second *Algol* Variable - 165
Opposition of *Eros* in 1900 - - - - - - - - - 244
A Photographic Search for an Intermercurial Planet - - - 322
Measurements of Photographic Intensities - - - - - 416
SCHUMANN, VICTOR. A Second Spectrum of Hydrogen Beyond λ 185 - - - - - - - - - - - - - - 312
SCHWARZSCHILD, K. On the Deviations from the Law of Reciprocity for Bromide of Silver Gelatine - - - - - - - - 89
On the Effect of Intermittent Exposure on Bromide of Silver Gelatine - - - - - - - - - - - - - 92
SEELIGER, H. A Remark on the Articles on the Density of the *Algol* Stars in the ASTROPHYSICAL JOURNAL, Vol. X, No. 5 - - 247
STONEY, G. JOHNSTONE. On the Escape of Gases from Planetary Atmospheres According to the Kinetic Theory - - 251–357
VOGEL, H. C. On the Progress Made in the Last Decade in Determination of Stellar Motions in the Line of Sight - - - 373
Description of the Spectrographs for the Great Refractor at Potsdam - - - - - - - - - - - - - - 393
WRIGHT, W. H. The Orbit of the Spectroscopic Binary χ *Draconis* - 131
The Velocity in the Line of Sight of ϵ *Leonis* - - - - - 414

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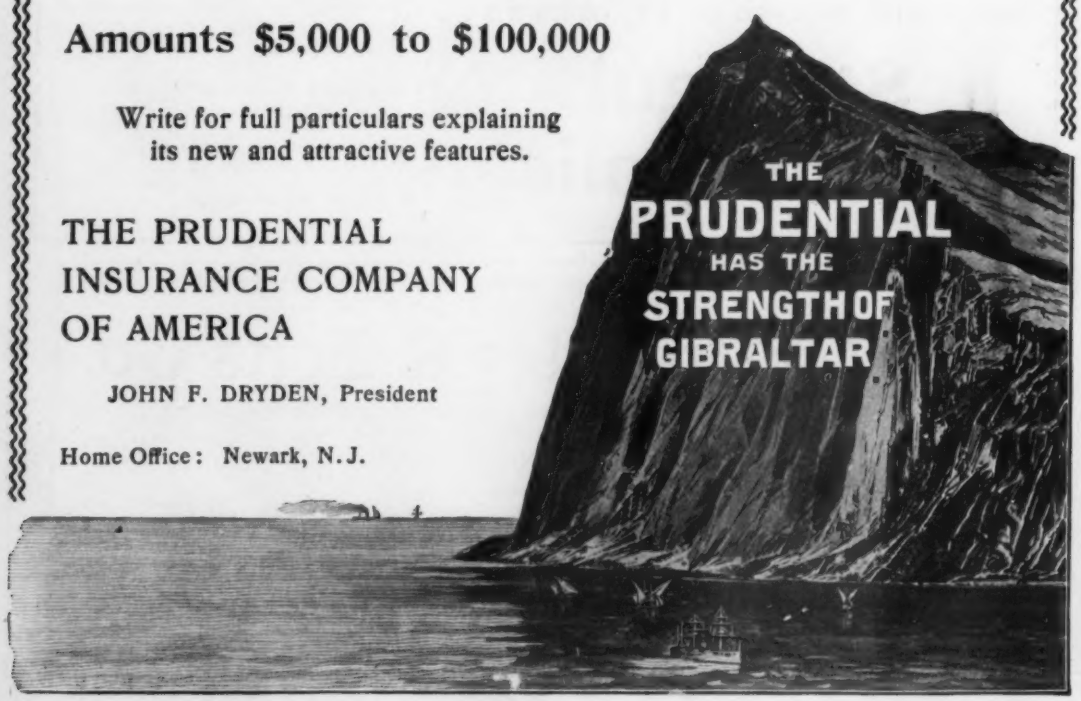

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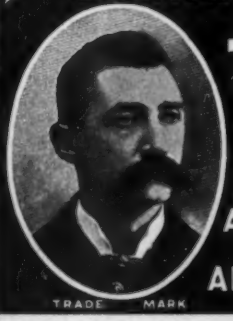

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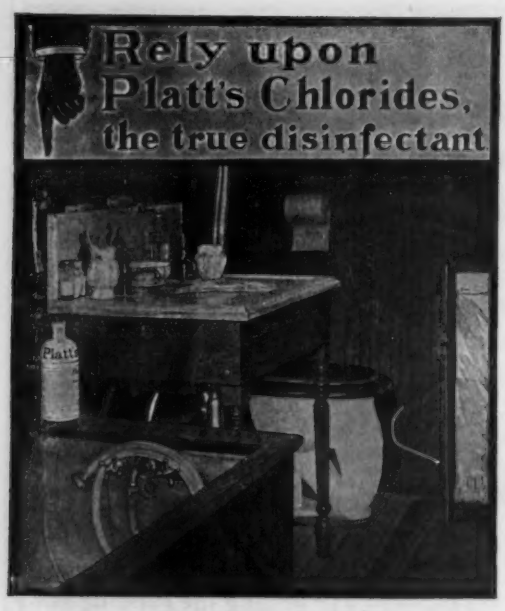

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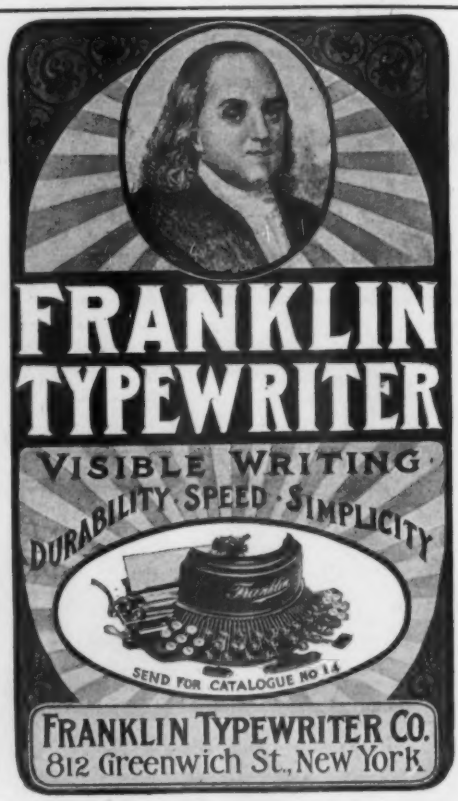

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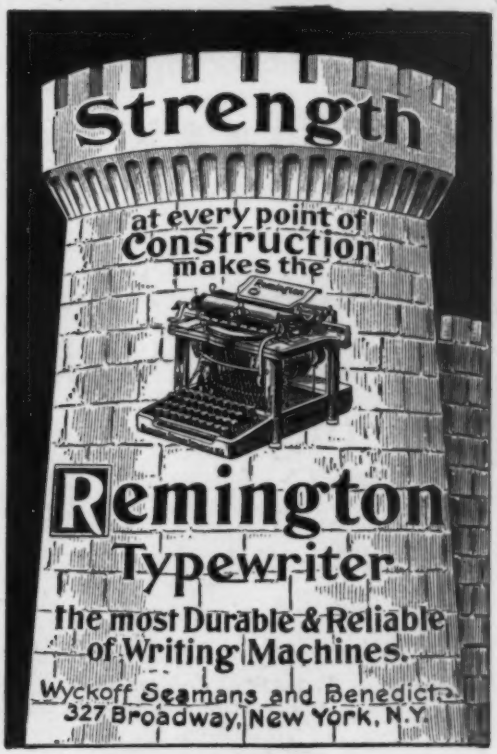

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